MULTISCALE STATISTICS OF LAGRANGIAN AND EULERIAN ACCELERATION IN TURBULENT STRATIFIED SHEAR FLOWS

Frank Jacobitz[1], Kai Schneider[2] & Marie Farge[3]
[1]*Shiley-Marcos School of Engineering, University of San Diego, 5998 Alcalá Park, San Diego, CA 92110, USA*
[2]*M2P2-CNRS, Aix-Marseille Université, 38 rue Joliot-Curie, 13451 Marseille Cedex 20, France*
[3]*LMD-IPSL-CNRS, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 5, France*

Abstract Direct numerical simulation data of homogeneous turbulence with shear and stratification are analyzed to study the Lagrangian and Eulerian acceleration statistics. Richardson numbers from $Ri = 0$, corresponding to unstratified shear flow, to $Ri = 1$, corresponding to strongly stratified shear flow, are considered. The scale dependence of the acceleration statistics is studied using a wavelet-based approach. The probability density functions (pdfs) of both Lagrangian and Eulerian accelerations exhibit a strong and similar influence on Ri. The extreme values for Lagrangian acceleration are weaker than those observed for the Eulerian acceleration. Similarly, the Lagrangian time-rate of change of fluctuating density is observed to have smaller extreme values than that of the Eulerian time-rate of change. Thus the time-rate of change of fluctuating density obtained at a fixed location is mainly due to advection of fluctuating density through this location. In contrast the time-rate of change of fluctuating density following a fluid particle is substantially smaller, and due to production and dissipation of fluctuating density.

INTRODUCTION

Properties of the Lagrangian acceleration of a fluid particle in turbulent flows are of fundamental importance. Recent studies range from theoretical investigations (e.g. Tsinober [14]) to applications such as the modeling of particle dispersion (e.g. Pope [10]). Both experimental (e.g. La Porta et al. [8]) as well as computational (e.g. Yeung [15] or Toschi and Bodenschatz [13]) approaches are employed. Previous investigations focused mostly on Lagrangian properties of isotropic turbulent flows. The Lagrangian acceleration was found to be strongly intermittent and heavy tails were observed in its pdf [8, 13].

Geophysical flows are often characterized by the presence of shear and stratification. Homogeneous turbulent stratified shear flow with constant vertical stratification rate $S_\rho = \partial\varrho/\partial y$ and constant vertical shear rate $S = \partial U/\partial y$ represents the simplest flow configuration in order to study the competing effects of shear and stratification. This flow has been investigated extensively in the past: Experimental studies include Komori et al. [7], Rohr et al. [12], Piccirillo and Van Atta [9], and Keller and Van Atta [6]. Numerical simulations include the work by Gerz et al. [1], Holt et al. [2], Jacobitz et al. [3], and Jacobitz [4].

The goal of this work is to investigate the acceleration statistics in turbulent stratified shear flows using direct numerical simulation. The equations of motion are transformed into a frame of reference moving with the mean velocity [11]. This transformation enables the application of periodic boundary conditions for the fluctuating components of velocity and a spectral collocation method is used for the spatial discretization. The solution is advanced in time with a fourth-order Runge–Kutta scheme. The simulations are performed on a parallel computer using $256 \times 256 \times 256$ grid points. Both the mean shear rate $S = \partial U/\partial y$ and the mean stratification rate $S_\rho = \partial\varrho/\partial y$ are constant. The primary non-dimensional parameter, the Richardson number $Ri = N^2/S^2$, where N is the Brunt-Väisälä frequency with $N^2 = -g/\varrho_0 S_\rho$, is varied from $Ri = 0$, corresponding to unstratified shear flow, to $Ri = 1$, corresponding to strongly stratified shear flow. The initial conditions are taken from a separate simulation of isotropic turbulence without density fluctuations, which was allowed to develop for approximately one eddy turnover time. The initial values of the Taylor-microscale Reynolds number $Re_\lambda = 56$ and the shear number $SK/\epsilon = 2$ are fixed.

NUMERICAL RESULTS

Details on the DNS data can be found in Jacobitz et al. [3]. We just recall that as the Richardson number is increased, the evolution of the turbulent kinetic energy changes from growth to decay at a critical value of $Ri_{cr} \approx 0.15$. The potential energy initially grows due to an increasing stratification rate with increasing Ri. Eventually, however, the decay of K also affects the evolution of K_ρ for large Richardson numbers. The Lagrangian and Eulerian accelerations are defined as $\mathbf{a_L} = \frac{\partial \mathbf{u}}{\partial \mathbf{t}} + \mathbf{u} \cdot \nabla \mathbf{u}$ and $\mathbf{a_E} = \frac{\partial \mathbf{u}}{\partial \mathbf{t}}$, respectively. This definition of the Lagrangian acceleration implies the perspective of an observer traveling with a fluid particle and the effects of shear and stratification are considered to be external forces. In the following, the accelerations are analyzed at time instant $St = 5$.

Figure 1 shows the probability distribution functions (pdfs) of the Lagrangian acceleration $\mathbf{a_L} = \mathbf{du/dt} = \partial\mathbf{u}/\partial\mathbf{t} + \mathbf{u}\cdot\nabla\mathbf{u}$ (left) and of the Eulerian acceleration $\mathbf{a_E} = \partial\mathbf{u}/\partial\mathbf{t}$ (right). The pdfs of both the Lagrangian and Eulerian accelerations have stretched-exponential shapes and they exhibit a strong and similar influence on the Richardson number Ri. It is also found that the extreme values of the Eulerian acceleration are above those of the Lagrangian acceleration, which is consistent with previous observations for sheared and rotating turbulence [5]. The variance of $\mathbf{a_E}$ was found to be larger than the variance of $\mathbf{a_L}$ for all cases considered in this study (not shown here). In addition, the flatness of $\mathbf{a_E}$ was observed to be larger than that of $\mathbf{a_L}$ and both flatness values decrease with increasing Richardson number (not shown here).

The time-rate of change of fluctuating density can also be considered using Lagrangian and Eulerian approaches. Figure 2 shows the pdfs of the Lagrangian time-rate of change of fluctuating density $d\rho/dt = \partial\rho/\partial t + \mathbf{u} \cdot \nabla\rho$ (left) and of the Eulerian time-rate of change $\partial\rho/\partial t$ (right). The difference in the pdfs of the time-rates of change is more pronounced than the difference obtained for the accelerations. The extreme values of the Eulerian time-rate of change of fluctuating density are substantially larger than those of the Lagrangian time-rate of change. This large difference is due to the nonlinear term in the affection-diffusion equation for fluctuating density and it is hence related to affection of fluctuating density. In other words, the time-rate of change of fluctuating density obtained at a fixed location by an Eulerian observer is mainly due to advection of density through this location, while the time-rate of change of fluctuating density observed by a Lagrangian observer following a fluid particle is substantially smaller and due to production and dissipation of fluctuating density.

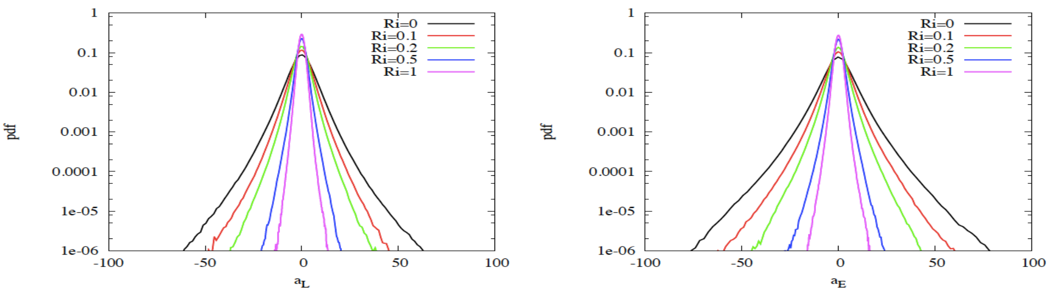

Figure 1. Pdfs of Lagrangian acceleration $\mathbf{a_L}$ (left) and Eulerian acceleration $\mathbf{a_E}$ (right).

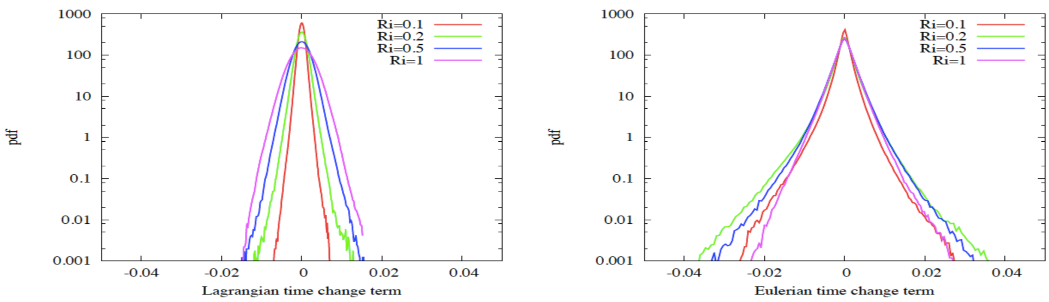

Figure 2. Pdfs of Lagrangian time-change of density $d\rho/dt$ (left) and Eulerian time-change of density $\partial\rho/\partial t$ (right).

References

[1] T. Gerz, U. Schumann, and S.E. Elghobashi. Direct numerical simulation of stratified homogeneous turbulent shear flows. *J. Fluid Mech.*, **200**, 563–594, 1989.

[2] S.E. Holt, J.R. Koseff, and J.H. Ferziger. A numerical study of the evolution and structure of homogeneous stably stratified sheared turbulence. *J. Fluid Mech.*, **237**, 499–539, 1992.

[3] F.G. Jacobitz, S. Sarkar, and C.W. Van Atta. Direct numerical simulations of the turbulence evolution in a uniformly sheared and stably stratified flow. *J. Fluid Mech.*, **342**, 231–261, 1997.

[4] F.G. Jacobitz. A comparison of the turbulence evolution in a stratified fluid with vertical or horizontal shear. *J. Turbul.*, **3**, 55, 1–16, 2002.

[5] F.G. Jacobitz, K. Schneider, W.J.T. Bos, and M. Farge. On Multiscale Acceleration Statistics in Rotating and Sheared Homogeneous Turbulence. *Proceedings of the International Symposium on Turbulence and Shear Flow Phenomena*, Poitiers, France, 2013.

[6] K.H. Keller and C.W. Van Atta. An experimental investigation of the vertical temperature structure of homogeneous stratified shear turbulence. *J. Fluid Mech.*, **425**, 1–29, 2000.

[7] S. Komori, H. Ueda, F. Ogino, and T. Mizushina. Turbulence structure in stably stratified open-channel flow. *J. Fluid Mech.*, **130**, 13–26, 1983.

[8] A. La Porta, G. A. Voth, A. M. Crawford, J. Alexander, and E. Bodenschatz. Fluid particle accelerations in fully developed turbulence. *Nature*, **409**, 1017, 2001.

[9] P.S. Piccirillo and C.W. Van Atta. The evolution of a uniformly sheared thermally stratified turbulent flow. *J. Fluid Mech.*, **334**, 61–86, 1997.

[10] S.B. Pope. Lagrangian PDF methods for turbulent flows. *Annu. Rev. Fluid Mech.*, **26**, 23–63, 1994.

[11] R.S. Rogallo. Numerical experiments in homogeneous turbulence. NASA Report No. TM 81315, 1981.

[12] J.J. Rohr, E.C. Itsweire, K.N. Helland, and C.W. Van Atta. Growth and decay of turbulence in a stably stratified shear flow. *J. Fluid Mech.*, **195**, 77–111, 1988.

[13] F. Toschi and E. Bodenschatz. Lagrangian properties of particles in turbulence. *Annu. Rev. Fluid Mech.*, **41**, 375, 2009.

[14] A. Tsinober. *An informal introduction to turbulence.* Kluwer Academic Publishers, 2001.

[15] P. K. Yeung. Lagrangian investigations of turbulence. *Annu. Rev. Fluid Mech.*, **34**, 115, 2002.