Turbulence Modeling for Oscillatory Pipe Flow

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Abstract Oscillating pipe flows, with zero mean velocity, are common in engineering systems and particularly in Stirling-based engines and heat pumps, e.g. pulse-tube cryogenic coolers. These flows are particularly challenging to model because their flow state depends on both the Reynolds number and the dimensionless frequency or Womerlsey number. A central challenge in modeling these systems is that the Reynolds number varies greatly within the cycle, often crossing from laminar to turbulent flow regimes. Indeed, flows that have super-critical Reynolds numbers may pass between states a total of four times; two laminar-turbulent transitions and two turbulent-laminar transitions. Contrary to a steady flow in which the transition between the laminar and the turbulent regimes is affected only by the Reynolds number, for oscillating flow the transition is affected by a combination of Re_{os} and a dimensionless frequency-based number such as Womersley (Wo). The objective of this work is to develop and validate a computational method that solves the incompressible oscillating flow equations for different combinations of Re_{os} and Wo.

Based on the experimental investigations of Hino et al. [1] and Ohmi et al. [2], four different regimes of the flow were identified: (I) laminar; (II) distorted laminar ("weakly turbulent"); (III) conditionally turbulent and (IV) critically turbulent. The characteristic diagram of Reynolds number versus Womersley number is presented in Figure 1, and illustrates the demarcations between the above-mentioned regimes. There is general agreement that the empirical correlation:

$$Re_{os}^{\text{crit}} = k\sqrt{Wo} \tag{1}$$

between regions I, II and III holds where k takes on values between 400 and 780, based on the findings of different investigations.

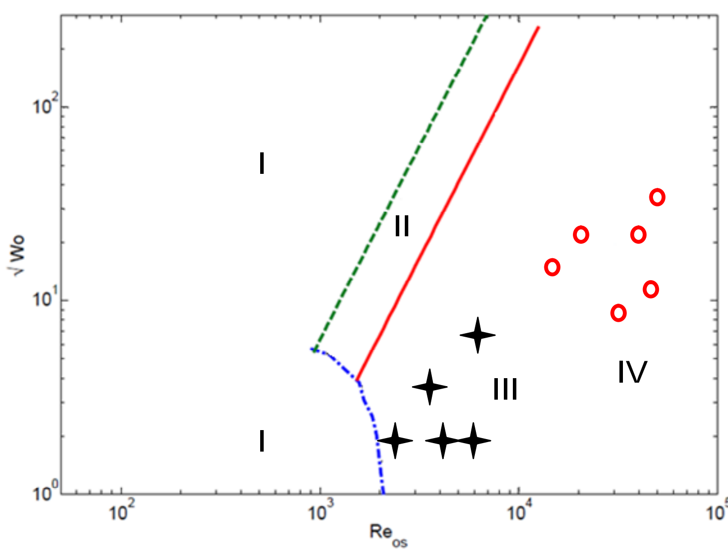

Figure 1. Characteristic diagram indicating different regimes of the flow, adapted from Ohmi et al. [2]. (I) laminar; (II) distorted laminar; (III) conditionally turbulent and (IV) critically turbulent. Red circles indicate the comparison cases used. Black Stars indicate the cases which Hino et al. [1] and Zhao et al. [3] defined as conditionally turbulent flow.

The numerical method utilized a Crank-Nicolson scheme on a non-uniform grid and different simple turbulence models were employed based on observed physics of the flow in all regimes, namely laminar, weakly turbulent, conditionally turbulent and critically turbulent. Computational results involving large-amplitude pulsating flows superimposed on a mean flow [4] showed that accurate wall-region modeling was disproportionately more important than that in the core flow. This can be explained by considering the integral form of mass conversation equation:

$$\frac{2}{a^2 U_{\max}} \int_0^a r u(r) \cdot \exp\left[-i\Phi(r)\right] dr = 1 \quad (2)$$

where $u(r)$ is the axial velocity, $\Phi(r)$ its associated phase-lag , r is the radial direction, and a is the pipe radius, and $U_{max}=\max[U(t)]$ where $U(t)$ is the bulk flow. Due to the axi-symmetric nature of the problem, mass conservations is weighted by the radius, emphasizing the importance of near-wall modeling. For example, then, an amplitude over-prediction in the near-wall region must be balanced by an under-prediction in the core region to satisfy mass conservation. Therefore, failure to adequately model the viscous sub-layer will have a detrimental effect on the predictions near the center of the pipe, or core flow.

Transition to turbulence was modeled phenomenologically; namely by initiating and terminating the turbulent viscosity in the decelerating phase ($dU(t)/dt<0$ and $Re_{os} > Re_{os}^{crit}$) for the conditionally turbulent model and in the accelerating phase and decelerating phases ($Re_{os} > Re_{os}^{crit}$) for the critically turbulent model). Figure 2 presents a comparison between the data provided of Ohmi et al. [2] and preliminary computational results of the velocity amplitude ratio (A) and the phase shift (•) as a function of normalized pipe radius employing different turbulent models. The comparison contains a fully turbulent computation with and without a near-wall modification, as well as conditionally and critically turbulent models.

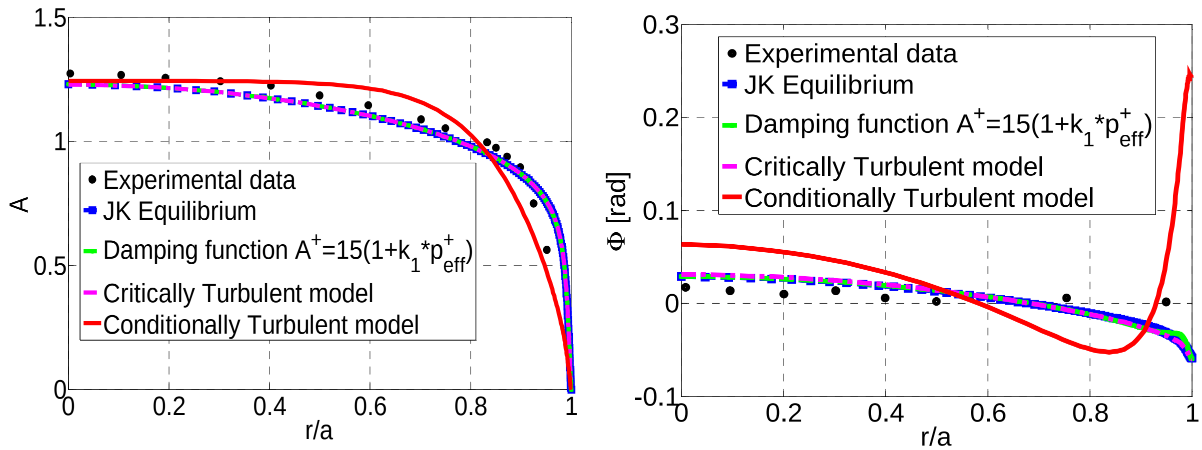

Figure 2. Velocity amplitude ratio and the phase shift comparison employing different turbulent models against the data of Ohmi [2]; Re_{os}=40,600, Wo=62.88

Apart from the conditionally turbulent model, all models show very similar results for the velocity amplitude. This is not surprising because under these conditions Re_{os}^{crit} takes on values between 3,170 and 6,185 and the flow is fully turbulent for the large majority of the cycle. For the conditionally turbulent model, transition to turbulence is only initiated when $dU(t)/dt<0$, corresponding to an instantaneous Reynolds number of 40,600. When comparing the models to the experimental data, it might be argued that the conditionally turbulent model produces comparable predictions near the wall and in the core region. However, this is not reflected in the phase-shift results. Nevertheless, insufficient near wall data was available to draw and unequivocal conclusion. Use of the damping function, employed here in conjunction with a lag-equation like that of Mao and Hanratty [5], did not show a meaningful improvement. A summary of comparisons (not shown, see red circles in Figure 1) indicated that in order to evaluate the most appropriate turbulence models, the consideration of different experimental conditions is needed; particularly in the interfaces between regimes II, III and IV.

References

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