IS ISOTROPY RESTORED AT SMALL SCALES IN FREELY DECAYING STRONGLY STRATIFIED TURBULENCE ?

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Abstract We analyse the scale-dependent anisotropy of homogeneous stratified turbulence. The Ozmidov scale ℓ_N [3] helps to compare the relative effects of inertia and of the buoyancy force, and thus to quantify the rise of anisotropy in different scale ranges: at large scales $l \gg \ell_N$ the anisotropy due to strong stratification is dominant, whereas at small scales $l \ll \ell_N$, universal 3D isotropic characteristics of turbulence appear to be restored. We investigate the corresponding dynamics using Direct Numerical Simulations (DNS) in freely decaying turbulence at different stratification rates. We confirm the return to isotropy of the small scales by analyzing the orientation-dependent power spectrum and poloidal/toroidal/density energy modes. To some extent, many characteristics of isotropic universality are restored at small scales but, surprisingly, the density spectrum (also potential energy spectrum) plays a particular role.

CONTEXT

In oceanic, atmospheric or engineering flows, turbulence can be strongly affected by stratification. In contrast with isotropic turbulence, anisotropic structures emerge in stably stratified turbulence: quasi-horizontal structures appear to be organized in vertically sheared layers. The thickness of these layers seems to scale according to an $O(1)$ magnitude of a related Froude number (see *e.g.* [1, 2]). The question is whether this anisotropy, characteristic of stratified turbulence, is present at all scales? To answer this question and provide a refined measure of the anisotropisation of the flow, Ozmidov [3] introduced a dedicated length scale ℓ_N which relies on the equilibrium between inertial and buoyancy forces. This length also yields a wavenumber $k_N \simeq 2\pi/\ell_N$ which can be computed as

$$k_N = \left(\frac{N^3}{\varepsilon}\right)^{1/2}$$

where N is the Brunt-Väisälä frequency —based on the stratification gradient—, and ε the kinetic energy dissipation. At large scales $k \ll k_N$ buoyancy is dominant and induces significant anisotropy, whereas at small scales $k_N \ll k$ inertia and, eventually, viscous forces, prevail and isotropic features are expected to be recovered.
The Ozmidov scale is widely used for analyzing data of numerical simulations [4, 5, 6] or of experiments [7, 8]. Such a refined description is thus important for an accurate characterization of turbulent mixing in stratified flows. We propose here a parametric study of the scale-by-scale anisotropy using dedicated spectral statistics.

RESULT OF DETAILED SCALE-BY-SCALE ANALYSIS

Classically, to measure the energy by scale — or for each wave number in Fourier space —, one uses averages of energy over spheres of radius k, and thus averages out the anisotropic contents of the energy distribution. In the case of stratified turbulence with axisymetric statistics about the vertical axis of gravity, the distribution of energy is not equi-distributed over the spherical shell of radius k by contrast to isotropic turbulence. We characterise this non equi-distribution of kinetic energy and potential energy by introducing the angular dependence of the power spectrum [9]. In the case of discrete analysis in DNS, we decompose the sphere into several rings O_i (six rings in our simulation as shown on the sketch of figure 1) and we define the energy by ring:

$$E_c(k, O_i) = \frac{1}{m_k^i} \sum_{\mathbf{k} \in O_i} |\hat{\mathbf{u}}_\mathbf{k}|^2 \qquad \text{and} \qquad E_{pot}(k, O_i) = \frac{1}{m_k^i} \sum_{\mathbf{k} \in O_i} |\hat{\rho}_\mathbf{k}|^2$$

where E_c is kinetic energy spectrum, $\hat{\mathbf{u}}_\mathbf{k}$ is the Fourier velocity vector, E_{pot} is the potential energy spectrum, $\hat{\rho}_\mathbf{k}$ is the Fourier component of density and $m_k^i = (\pi/4)(\theta_i - \theta_{i+1})^{-1}(sin(\theta_i) - sin(\theta_{i+1})^{-1}$ is a normalization term (so that E_c recovers the isotropic scalings of classical Kolmogorov spectrum in absence of stratification).
We have performed simulations at four stratification intensities, thus at different Froude numbers ranging from about 0.13 to 1.06, with 2048^3 grid points, so that the Reynolds numbers are rather high, of order $Re \simeq$ 3000–4000. For instance, figure 1 shows the power spectra for four stratified rates. On each plot the Ozmidov wavenumber k_N is indicated to delimitate the large scale stratification-affected range from the smaller scale range.
We show that at Froude number of order one (the lowest stratification rate in our parametric study), on figure 1(a), large scales are clearly anisotropic (kinetic energy is concentrated towards the polar ring O_1), whereas at scales smaller than

the Ozmidov one (wavenumbers larger than k_N), the spectra join again, as a sign that 3D isotropy is very much recovered. At lower Froude number (higher stratification rate) on figures 1(b) and (c), the large scales also exhibit anisotropy, but in these two cases energy concentration in the polar ring O_1 is very strong down to the Ozmidov scale $k \leq k_N$, but is clearly diminished at smaller scales, although full isotropic recovery is not allowed by the reduced small scale range before viscous cut-off. Finally, at very high stratification rate on figure 1(d) (very low Froude number), all scales are strongly anistropic with at least one decade difference in the energy between horizontal motion (vertical wavenumbers in ring O_1) and vertical one (horizontal wavenumbers in ring O_6).

In addition, we analyse further the dynamics of the flow by decomposing the velocity into two modes : one is the toroidal part—the vortex mode containing all the vertical vorticity—, the other the poloidal part—motion related to waves in the linear regime. The Fourier velocity vector is thus split in a poloidal part, with kinetic energy E^P (mostly linked to vertical velocity) and toroidal part with kinetic energy E^T (linked to horizontal velocity) [9]. Our DNS simulations show that for large scales, $E^T \gg E^P$, and the poloidal part of kinetic energy spectrum is of the order of the density spectrum $E^P \sim E^\rho$ (potential energy), as expected from equipartition of kinetic and potential energy by internal gravity waves. At smaller scales close to the Ozmidov scale, DNS results show that $E^T \sim E^P \sim E^\rho$ as a sign of restoration of isotropy. Nevertheless, at very small scales below the Ozmidov scale, DNS always show that $E^T \sim E^P$ but surprisingly energy equipartition no longer holds : $E^\rho \gg E^T \sim E^P$. In our presentation, we will discuss more precisely this phenomenon and results of our parametric study, in relation with the flow structures in physical space.

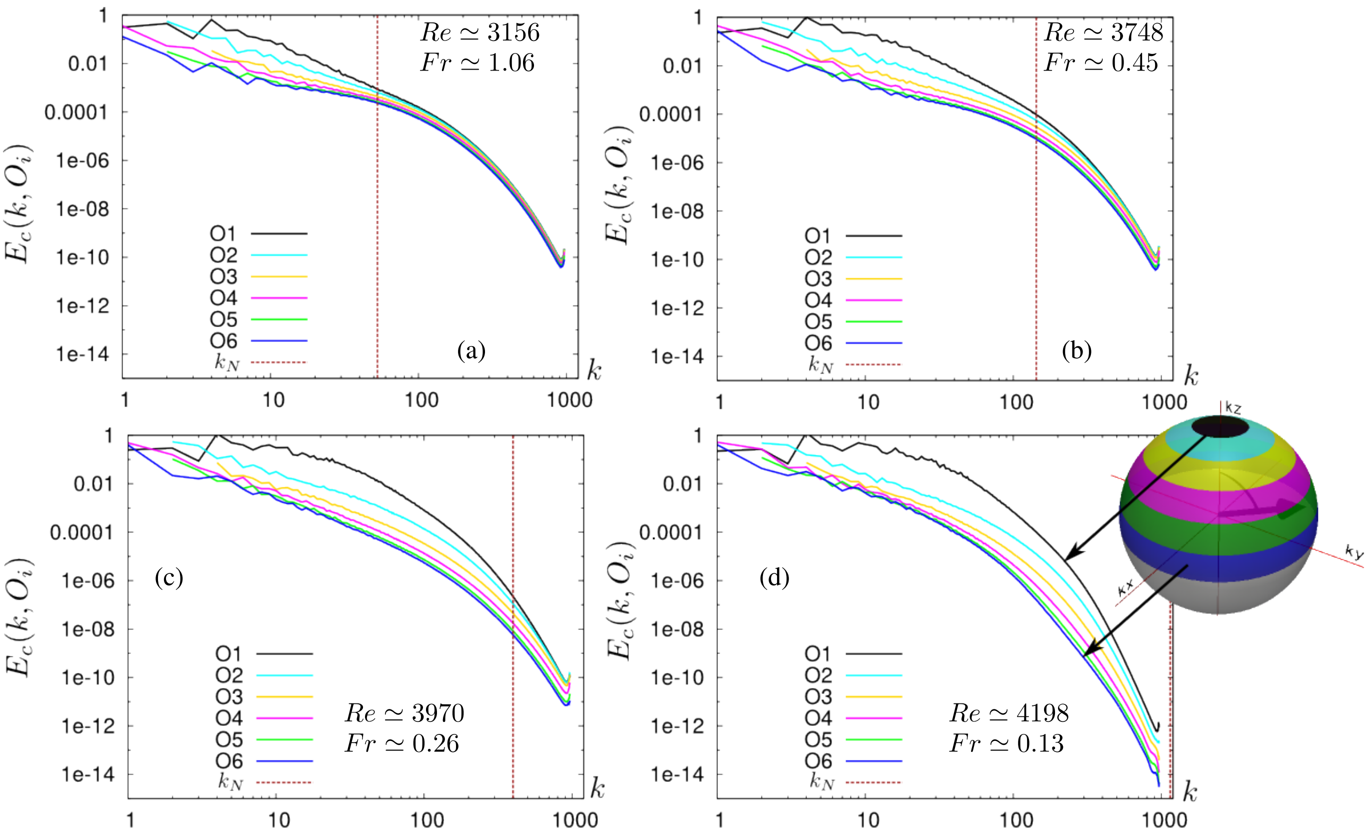

Figure 1. Angle-dependent power spectrum at increasing Froude numbers as indicated from (a) to (d) (different stratification rates), and sketch of the angular decomposition into six spectral rings.

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