

## VELOCITY-GRADIENT PROBABILITY DISTRIBUTION FUNCTIONS IN A LAGRANGIAN MODEL OF TURBULENCE

L. Moriconi<sup>1</sup>, R.M. Pereira<sup>2</sup> & L.S. Grigorio<sup>3</sup>

<sup>1</sup>*Instituto de Física, Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brazil*

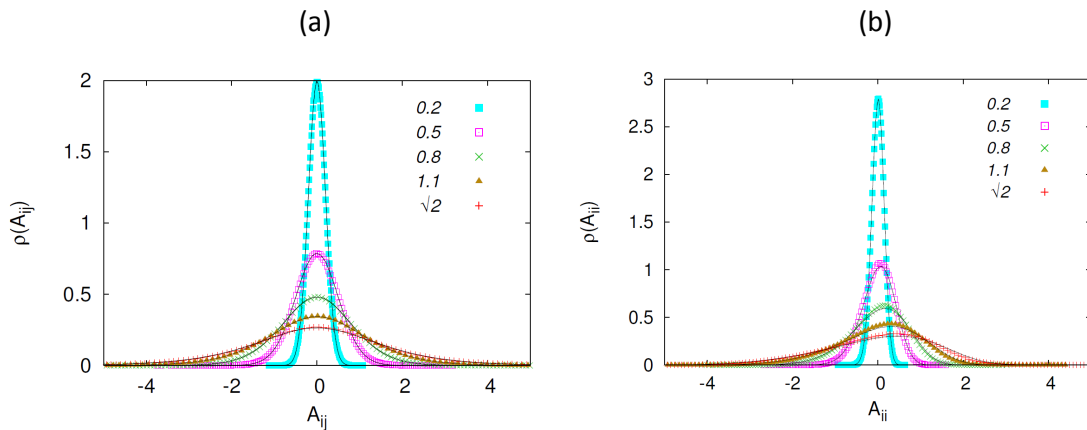
<sup>2</sup>*Laboratoire de Physique, École Normale Supérieure de Lyon, Lyon, France*

<sup>3</sup>*Centro Federal de Educação Tecnológica Celso Suckow da Fonseca, Nova Friburgo, Brazil*

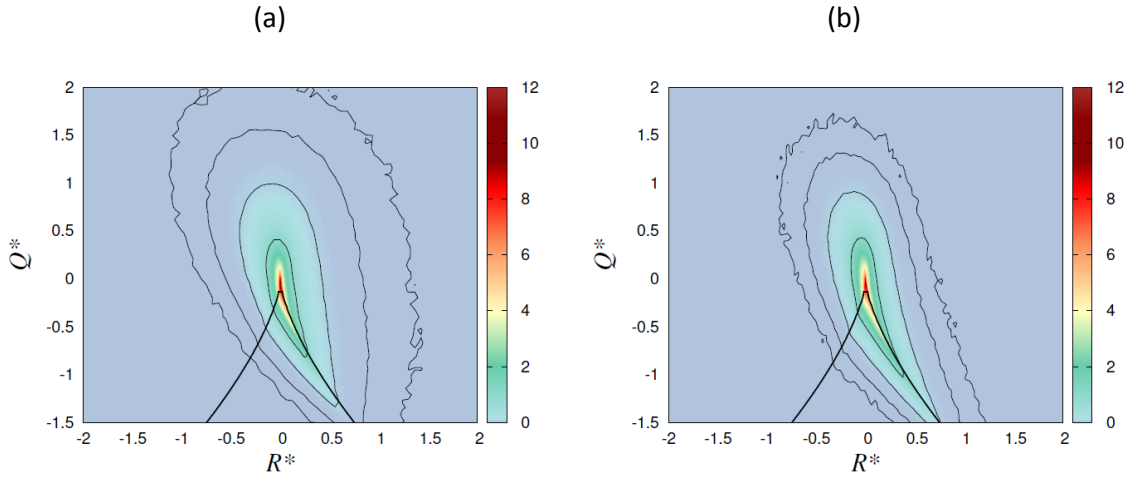
**Abstract** The Recent Fluid Deformation Closure (RFDC) model of lagrangian turbulence is recast in path-integral language within the framework of the Martin-Siggia-Rose functional formalism. In order to derive analytical expressions for the velocity-gradient probability distribution functions (vgPDFs), we carry out noise renormalization in the low-frequency regime and find approximate extrema for the Martin-Siggia-Rose effective action. We verify, with the help of Monte Carlo simulations, that the vgPDFs so obtained yield a close description of the single-point statistical features implied by the original RFDC stochastic differential equations.

It has been long known, since the seminal work of Batchelor and Townsend [1], that spatial derivatives of a turbulent velocity field do not behave as gaussian random variables. The current view on this still barely understood phenomenon is that the non-gaussian fluctuations of the velocity gradients (the hallmark of turbulent intermittency) are likely to be related to the existence of long-lived coherent structures and to deviations from the Kolmogorov “K41” scaling, both important ingredients in the contemporary phenomenology of turbulence [2]. Notwithstanding the notorious difficulties with first-principle theories of intermittency, it is actually possible to devise simplified fluid dynamical models that would capture relevant qualitative features of the intermittent fluctuations of the velocity gradient tensor [3].

We have carried out an analytical study of the vgPDFs in the RFDC lagrangian model of turbulence [4]. The Martin-Siggia-Rose (MSR) framework [5] in its path-integral formulation proves to be a very convenient setup, where standard field-theoretical semiclassical approaches can be straightforwardly applied. Once it is difficult to establish exact saddle-point solutions for the Euler-Lagrange equations associated either to the bare or to the effective MSR action, we have used, as an approximation, solutions that hold in the regime of small noise strength. A further source of technical difficulty is related to the precise evaluation of the effective MSR action up to one-loop order: in fact, one should take into account a large number of vertex corrections, leading to non-local kernels as the result of much more involved computations. We have, thus, put forward a pragmatistical strategy for the evaluation of the effective MSR action where, as working hypotheses, (i) only the noise vertex is corrected up to one-loop order and (ii) a low-frequency approximation for the renormalized noise vertex is implemented. Nevertheless the above simplifying assumptions, the resulting analytical PDFs turn out to be satisfactorily compared, as indicated in Figs. 1 and 2, to the empirical ones for a meaningful range of bare noise coupling constants [6].



**Figure 1.** Comparative linear plots of vgPDFs, for the non-diagonal (a) and diagonal (b) components of the velocity gradient tensor. Symbols refer to empirical PDFs obtained from numerical solutions of the RFDC modelling stochastic differential equations at various external forcing strengths, while solid lines are analytical PDFs derived from path-integral evaluations. The fittings in (a) and (b) are very reasonable within about two standard deviations around the peak values of the vgPDFs.



**Figure 2.** Contour plots for the joint PDFs of the normalized Cayley-Hamilton invariants  $Q^*$  and  $R^*$  for a certain Reynolds number and stochastic forcing strength. The level curves have PDF values equal to 1,  $10^{-1}$ ,  $10^{-2}$  and  $10^{-3}$ . Figure (a) is obtained from the direct numerical integration of the RFDC modelling stochastic equations, while figure (b) is evaluated from the analytical vgPDF derived from path-integral evaluations. The inverted V-shaped lines in both figures (a) and (b) indicate the Vieillefosse zero discriminant line.

## References

- [1] G.K. Batchelor and A.A. Townsend, (1949), The Nature of Turbulent Motion at Large Wave-Numbers, *Proc. R. Soc. Lond. A*, vol. 199, pp. 238-255.
- [2] U. Frisch, (1995): *Turbulence: The Legacy of A.N. Kolmogorov*, Cambridge University Press; P.A. Davidson, (2004): *Turbulence, an Introduction for Scientists and Engineers*, Oxford University Press.
- [3] C. Meneveau, (2011), Lagrangian Dynamics and Models of the Velocity Gradient Tensor in Turbulent Flows, *Ann. Rev. Fluid Mech.*, vol. 43, pp. 219-245.
- [4] L. Chevillard and C. Meneveau, (2006), Lagrangian dynamics and statistical geometric structure of turbulence, *Phys. Rev. Lett.*, vol. 97, pp. 174501-174504.
- [5] P.C. Martin, E.D. Siggia and H.A. Rose, (1973), Statistical Dynamics of Classical Systems, *Phys. Rev. Lett. A*, vol. 8, pp. 423-437.
- [6] L. Moriconi, R.M. Pereira, and L.S. Grigorio, (2014), Velocity-Gradient Probability Distribution Functions in a Lagrangian Model of Turbulence, *J. Stat. Mec.*, vol. 10, P10015.