DNS OF INERTIAL WAVE ATTRACTORS IN A LIBRATING ANNULUS WITH HEIGHT-DEPENDENT GAP WIDTH

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Abstract Direct numerical simulations (DNS) of inertial wave attractors have been carried out in a librating Taylor-Couette system with broken mirror symmetry in the radial-axial cross-section. The inertial wave excitation mechanism and its localisation at the edges was clarified by applying boundary layer theory. Additional resonance peaks in the simulated response spectra were found to agree with low-order wave attractors obtained by geometric ray tracing. Numerics and theory are in qualitative agreement with recent lab experiments.

INTRODUCTION

Rotation Ω_0 is one of the most important system parameters in geophysical fluid dynamics (GFD) due to stratification of angular momentum and the Coriolis force in the co-rotating frame of reference. Small perturbations can excite inertial waves which may take part in the redistribution of kinetic energy and angular momentum. Inertial waves are shear waves that are band-limited to frequencies $0 < \sigma < 2\Omega_0$ and dispersive with respect to direction. Complex system responses are possible so that it is of fundamental interest to understand inertial waves and related interactions in order to quantify their relevance for applications (see, e.g., [2] and references therein).

CONFIGURATION AND NUMERICAL MODEL

We consider the Taylor–Couette system shown in Figure 1. Rigid lids close the container at the top and bottom. The vessel is of height h, has height dependent radii $r_1(z) < r_2 \sim O(h)$, and is rotating with a mean angular velocity Ω_0 around its axis of symmetry. It is filled with an incompressible Newtonian fluid of kinematic viscosity ν so that Ekman numbers down to $E = \nu\Omega_0^{-1}r_1^{-2}(0) \simeq 10^{-6}$ can be accessed by DNS. In order to study focusing, wave attractors, and the effect of a particular inclination angle, the rectangular symmetry in the radial-axial plane has been broken by replacing the inner cylinder with a frustum of apex half-angle $\alpha \approx 5.7°$. Here inertial waves are excited by longitudinal librations of different parts of the walls. That is, the mean rotation rate is modulated by harmonic oscillations: $\Omega(t) = \Omega_0(1 + \varepsilon \sin \omega t)$, where ω denotes the frequency and $0 < \varepsilon < 1$ the amplitude of libration. We consider either the outer cylinder together with the lids in libration and the frustum fixed in the co-rotating frame ($\Omega_2 = \Omega(t)$, $\Omega_1 = \Omega_0$) or *vice versa*, focusing on the weakly non-linear regimes with Rossby numbers $R \sim \varepsilon = O(10^{-1})$.

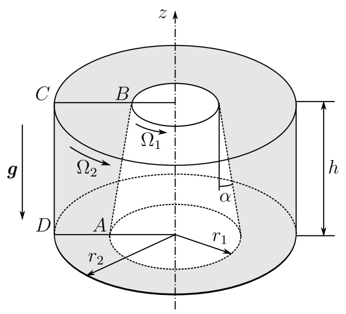

Figure 1. Sketch of the librating annular cavity with an inner frustum.

In order to resolve the wave attractors and the different wave excitation processes within the numerical model, two- and three-dimensional DNS of the set-up have been performed. We used a code that solves the incompressible Navier–Stokes equations for the volume flux components in generalized curvilinear coordinates. The numerical scheme conserves mass, momentum and kinetic energy when the grid fulfils local orthogonality in a discrete sense.

RESULTS

We show that wave excitation is coherent and localised, i.e., strongly bound to the driving boundary layer flow. Waves excited at the edges of the annulus (inner edges in Figure 2, left) are due to a discrepancy in the wall-tangential mass flux in the boundary layer across an edge. This is similar in other geometries with discontinuities. Our results suggest that inertial waves are easily excited in the ocean anywhere the bottom's slope changes abruptly as at a continental shelf [3]. In the annular cavity inertial wave attractors manifest as closed-loop patterns of internal shear layers; cf. Figure 2 [left]. Kinetic energy corresponding to a particular wave frequency is focused on the wave attractor and may thus provide loci of non-linear interactions.

The two different configurations investigated can be clearly distinguished by their response spectra; cf. Figure 2 [right]. Baselines in the spectra are related to Ekman pumping/suction. Over the librating frustum an f-plane approximation suggests presence of an oscillatory Ekman layer with resonance frequency $f_*/\Omega_0 = 2\sin\alpha \approx 0.2$ at which Ekman pumping/suction reaches its maximum. Over the librating lids an oscillatory Ekman layer with resonance frequency $f/\Omega_0 = 2$ is established, whereas the Stokes layer over the librating outer cylinder 'erupts' for $\omega \to 0$ but excites waves rather inefficiently. The narrow resonance peaks in Figure 2 [right] cannot be explained by boundary layer theory. Flow patterns and frequency intervals correspond to low-order wave attractors or neutral orbits found by geometric ray tracing (shaded areas in Figure 2, right; cf. [1]). The 'quality factors' of these peaks are not yet very well understood.

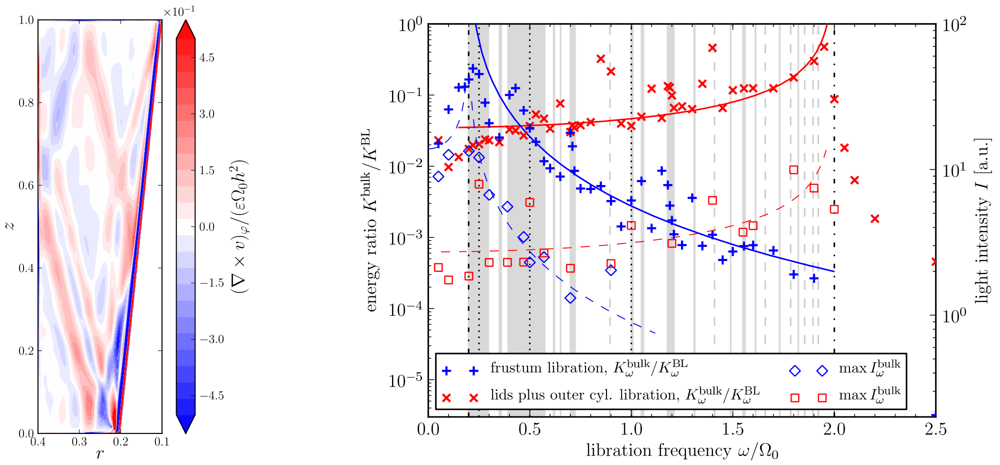

Figure 2. [left] Axial-radial cross section through the simulated axisymmetric $1/1$ wave attractor due to frustum librations for $\omega/\Omega_0 = 0.47$, $R_1 = \varepsilon = 0.2$, $E = 3.18 \times 10^{-5}$. Inertial waves are visualized by contours of the azimuthal vorticity component. [right] Wave excitation efficiency spectra due to different walls in libration measured by the integral kinetic energy ratio $K^{\rm bulk}_\omega/K^{\rm BL}_\omega$ $(+, \times)$ obtained from Fourier-filtered velocities in the bulk and the boundary layer (BL). Parameters as before, but for similar libration Rossby numbers $R_{1,2} = \varepsilon[r_{1,2}(0)/r_1(0)] = 0.2$. Lab measurements (light intensities I; $\Diamond$, $\Box$) and theoretical scalings (—, - -) are overlayed for comparison. Shaded areas indicate the low-order wave attractor intervals and neutral orbits found by ray tracing (after [1]).

OUTLINE

In the contribution the numerical model will be presented in more detail. The inertial wave excitation mechanism will be discussed and supported by DNS data. Flow patterns predicted by geometric ray tracing and lab measurements will be shown for selected cases. Both wave excitation and wave attractors will be related to the response spectra. Depending on the progress of the ongoing research, spectra for kinetic energy, dissipation, helicity, and torque will be be shown. Relevant integral quantities and their balances will be discussed in order to elucidate the resonance condition.

References

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