THE STREAMWISE TURBULENCE INTENSITY IN THE INTERMEDIATE LAYER OF HIGH REYNOLDS TURBULENT PIPE FLOW

J.C. Vassilicos[1,3], J.-P. Laval[2], J.-M. Foucaut[3] and M. Stanislas[3]
[1] *Department of Aeronautics, Imperial College London, London SW7 2AZ, United Kingdom*
[2] *CNRS, UMR 8107, F-59650 Villeneuve d'Ascq, France*
[3] *ECLille, LML, F-59650 Villeneuve d'Ascq, France*

Abstract A modification of the Townsend-Perry attached eddy model is derived in order to reproduce a more realistic variation of the integral length scale. A new wavenumber range is introduced to the model at wavenumbers smaller than the Townsend-Perry k^{-1} spectrum. This necessary addition can also account for the high Reynolds number outer peak of the turbulent kinetic energy in the intermediate layer. An analytic expression is obtained for this outer peak in agreement with extremely high Reynolds number data by Hultmark *et al.* [3, 4]. The finding of Dallas *et al.* [1] that it is the eddy turnover time and not the mean flow gradient which scales with distance to the wall and skin friction velocity in the intermediate layer implies, when combined with Townsend's (1976) production-dissipation balance, that the mean flow gradient has an outer peak at the same location as the turbulent kinetic energy.

INTRODUCTION

Townsend developed a well-known attached-eddy model [6] for pipe/channel and turbulent boundary layer (TBL) flows to predict the profile with distance from the wall of the turbulent kinetic energy (TKE). For wall distances much larger than the wall unit δ_ν and much smaller than, say, pipe radius δ, the turbulent kinetic energy scales with the square of the wall friction velocity u_τ and decreases logarithmically with distance to the wall. However, measurements in turbulent boundary layers dating from about twenty years ago (see [2]) as well as more recent Nano Scale Thermal Anemometry Probe (NSTAP) data obtained in the Princeton Superpipe [4]) show that an outer peak appears in the mean square fluctuating streamwise velocity at distances from the wall between about $100\delta_\nu$ and $800\delta_\nu$ when the turbulent Reynolds number $Re_\tau = \delta/\delta_\nu$ is large enough. Such non-monotonic behaviour in regions where the mean flow is monotonically increasing is hard to account for in current turbulence models and theory. Starting with the spectral model of [5] there have been numerous developments and extensions of the attached eddy model but none has accounted for the outer peak in turbulent kinetic energy. Moreover, this model predicts that the integral-scale varies logarithmically with wall distance, which is too weak to be in agreement with several observations. The only way to repair this model without removing its attached eddy part is to introduce a fourth large scale range in the model.

MODEL AND RESULTS

We consider a model of the streamwise energy spectrum $E_{11}(k_1, y)$ with the following four ranges (see [7] for more details): (i) $k_1 < 1/\delta_\infty$ where $E_{11}(k_1) \approx C_\infty u_\tau^2 \delta$ with a constant C_∞ independent of wavenumber; (ii) $1/\delta_\infty < k_1 < 1/\delta_*$ where $E_{11}(k_1) \approx C_1 u_\tau^2 \delta (k_1\delta)^{-m}$ where $0 < m < 1$ and C_1 is also a constant independent of wavenumber; (iii) $1/\delta_* < k_1 < 1/y$ where $E_{11}(k_1) \approx C_0 u_\tau^2 k_1^{-1}$ where C_0 is a constant independent of wavenumber, y and Re_τ (the 'attached eddy' range); (iv) $1/y < k_1$ where $E_{11}(k_1)$ has the Kolmogorov form $E_{11}(k_1, y) \sim \epsilon^{2/3} k_1^{-5/3} g_K(k_1 y, k_1\eta)$. Compared to the Perry-Towsend model, two new scales δ_∞ and δ_* are introduced to bound the new wavenumber range which must grow as the position y where $E_{11}(k_1, y)$ is evaluated approaches the wall and distances itself from the centre of the pipe within $\delta_\nu \ll y \ll \delta$. As δ_∞/δ_* must depend on $y^+ = y/\delta_\nu$ and Re_τ a plausible functional dependence is

$$\delta_\infty/\delta_* \approx A\,(y/\delta)^{-p} Re_\tau^{-q} \approx A\,(y^+)^{-p} Re_\tau^{p-q} \tag{1}$$

where A is a dimensionless constant and where the physics impose $p, q > 0$ and $p > q$. Matching the energy spectral forms at the two bounds of our new range and assuming again a power law form for $\delta_*/\delta = B(y/\delta)^\alpha Re_\tau^\beta$, an integration of $E_{11}(k_1)$ yields

$$\frac{1}{2}\overline{u'^2}(y)/u_\tau^2 \approx C_{s0} - C_{s1}\ln(\delta/y) - C_{s2}\,(y/\delta)^{p(1-m)} Re_\tau^{q(1-m)} \tag{2}$$

in $\delta_\nu \ll y < y_*$ where C_{s0} is a weak function of Re_τ, C_{s1} and C_{s2} are independent of Re_τ and the upper bound y_* is deduced from a viability requirement of our four-range spectra. Our model recovers the original Twonsend-Perry formula but only in the range $y_* < y \ll \delta$. In Fig. 1 we show the result of the fit against the NSTAP superpipe data of the Townsend-Perry formula

$$\frac{1}{2}\overline{u'^2}(y)/u_\tau^2 \approx C_0 + C_0\ln(\delta/y) \tag{3}$$

in the range $y_* < y \ll \delta$ and our new formula (2) in $\delta_\nu \ll y < y_*$. Note the gradual development as Re_τ increases of the peak of turbulence intensity inside the intermediate region $\delta_\nu \ll y < y_*$. The position of this peak (y_{peak}) which

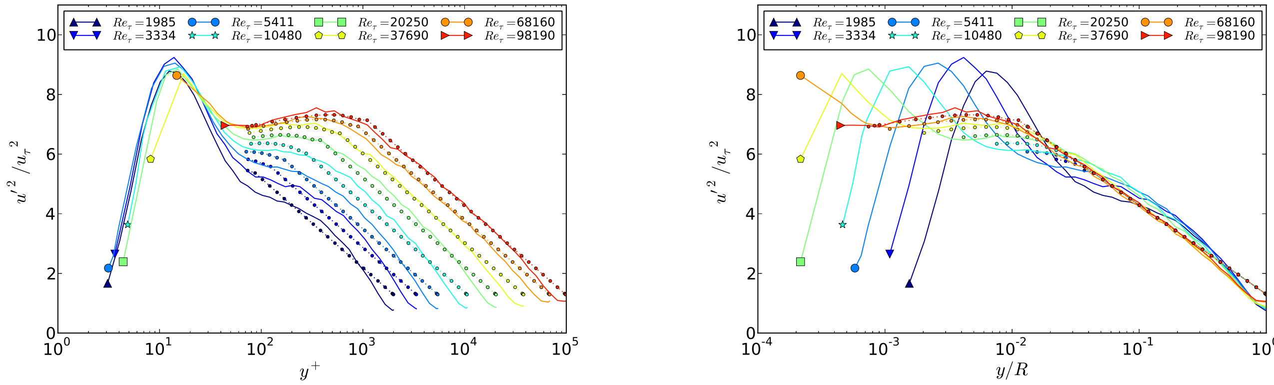

Figure 1. Plots of $\overline{u'^2}(y)/u_\tau^2$ versus y^+ (left) and y/δ (right) obtained from the NSTAP Superpipe data of [3, 4] for different values of Re_τ. The circles are calculated for all Reynolds numbers from equations (3) and (2) with $y_* = \delta A^{1/p} Re_\tau^{-q/p}$ and $A = 0.2$, $C_0 = 1.28$, $m = 0.37$, $q = 0.79$, $p = 2.38$ and $\alpha = 1.21$.

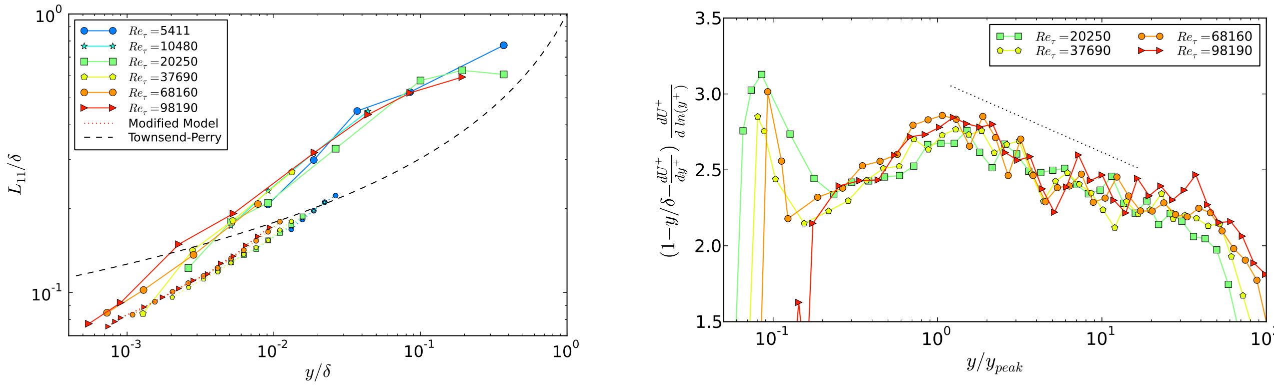

Figure 2. (Left) Normalised integral scales L_{11}/δ obtained from NSTAP Superpipe energy spectra plotted versus y/δ for various Reynolds numbers. Also plotted are the Townsend-Perry and our modified model's prediction for L_{11}/δ. (Right) Linear-logarithmic plot of $(1 - y/\delta - \frac{d\overline{u}^+}{dy^+})\frac{d\overline{u}^+}{d\ln y^+}$ versus y/y_{peak} for the largest values of Re_τ obtained from the NSTAP Superpipe mean flow data

starts appearing at Re_τ values larger than about 20,000 evolves, according to our model with the fitted parameters, as $y_{peak}/\delta_\nu \approx 0.23 Re_\tau^{0.67}$. This means, as observed in Fig. 1, that y_{peak}/δ decreases and y_{peak}/δ_ν increases with Re_τ as indeed observed. This modified model leads to a power law dependence in y of the integral scale for $y < y_*$ which is in better agreement with observations and with the NSTAP superpipe data than the original Townsend-Perry logarithmic dependence (see Fig. 2).

Moreover, the very high Re_τ Princeton Superpipe data used here support the view of [1] that it is the eddy turnover time $\tau \equiv E/\epsilon$ (E is the total TKE and ϵ its dissipation) that is independent of ν and δ in the range $\delta_\nu \ll y \ll \delta$ rather than the mean flow gradient. This implies $\tau \sim y/u_\tau$ in that range, a relation which can serve as a unifying principle across Reynolds numbers in turbulent pipe/channel flows. Assuming a production-dissipation balance in $y_{P\epsilon} < y \ll \delta$ (where $y_{P\epsilon}$ is smaller than y_{peak}), a profile for E^+ similar to that of $\overline{u'^2}/u_\tau^2$ and $-\langle u'v'\rangle \approx u_\tau^2$, it follows that $\frac{d\overline{u}^+}{d\ln y^+}$ (i) has an outer peak at the same position $y = y_{peak}$ where $\overline{u'^2}/u_\tau^2$ has an outer peak, and (ii) decreases with distance from the wall as a function of $\ln(\delta/y)$ where $y_* < y \ll \delta$. As seen in Fig. 2, both results are supported by the NSTAP data.

References

[1] V. Dallas, J.C. Vassilicos, and G.F. Hewitt. Stagnation point von Kármán coefficient. *Phys. Rev. E*, **80**:046306, 2009.

[2] H. H. Fernholdz and P. J. Finley. The incompressible zero-pressure-gradient boundary layer: an assessment of the data. *Prog. Aerospace Sci.*, **32**:245–311, 1996.

[3] M. Hultmark, M. Vallikivi, S. C. C. Bailey, and A. J. Smits. Turbulent pipe flow at extreme Reynolds numbers. *Phys. Rev. Lett.*, **108**:094501, 2012.

[4] M. Hultmark, M. Vallikivi, S. C. C. Bailey, and A. J. Smits. Logarithmic scaling of turbulence in smooth- and rough-wall pipe flow. *J. Fluid Mech.*, **728**:376–95, 2013.

[5] A. E. Perry, S. Henbest, and M. S. Chong. A theoritical and experimental study of wall turbulence. *J. Fluid Mech.*, **165**:163–199, 1986.

[6] A. A. Townsend. *The structure of turbulent shear flow (second edition)*. Cambridge university press, Cambridge, 1976.

[7] J.C. Vassilicos, J.-P. Laval, J.-M. Foucaut, and M. Stanislas. The streamwise turbulence intensity in the intermediate layer of turbulent pipe flow. *submitted to J. Fluid Mech. (arXiv:1411.7276)*, 2014.