ENERGY DISSIPATION AND FLUX LAWS FOR UNSTEADY TURBULENCE

Susumu Goto [1] & J. Christos Vassilicos[2]
[1] *Graduate School of Engineering Science, Osaka University*
1-3 Machikaneyama, Toyonaka, Osaka, 560-8531 Japan
[2] *Department of Aeronautics, Imperial College London*
London, SW7 2AZ, United Kingdom

Abstract Direct Numerical Simulations of spatially periodic unsteady turbulence show that the high Reynolds number scalings of the instantaneous energy dissipation rate and interscale energy flux at intermediate wavenumbers are qualitatively different from the well-known $u'(t)^3/L(t)$ cornerstone scalings of equilibrium turbulence where $u'(t)$ and $L(t)$ are time-dependent rms velocity and integral length-scales. Instead, they both scale as $U_0 L_0\, u'(t)^2/L(t)^2$ where L_0 and U_0 are length and velocity scales characterizing initial/overall unsteady turbulence conditions.

INTRODUCTION

Recent wind and water tunnel experiments (see Vassilicos 2015 Ann. Rev. Fluid Mech. 47, 95-114) show that in a variety of decaying turbulent flows with different levels of statistical homogeneity and well-defined $k^{-5/3}$ wavenumber dependence of the energy spectrum $E(k,t)$, the following high Reynolds number law of the dissipation rate $\epsilon(t)$ of turbulent kinetic energy is observed:

$$\epsilon(t) \sim \frac{\sqrt{Re_0}}{Re_\lambda(t)} \frac{u'(t)^3}{L(t)} \sim \nu Re_0 \frac{u'(t)^2}{L(t)^2} \tag{1}$$

where t is a time surrogate for streamwise distance, $L(t)$ is an integral length-scale, $Re_0 = U_0 L_0/\nu$ is a global Reynolds number based on a velocity U_0 and a length scale L_0 characterizing the initial/inlet conditions, $Re_\lambda = u'\lambda/\nu$ is a local Reynolds number based on the Taylor length $\lambda(t)$ and the rms turbulence velocity $u'(t)$, and ν is the kinematic viscosity of the fluid. This dissipation law is fundamentally different from the well-known scaling first introduced by Taylor (1953, Proc. R. Soc. Lond. A 151, 421-444)

$$\epsilon(t) = C_\epsilon \frac{u'(t)^3}{L(t)}\,, \tag{2}$$

where C_ϵ is a dimensionless constant. This law (2) of dissipation relates $\epsilon(t)$, which is a small-scale quantity, to the large-scale flow properties $L(t)$ and $u'(t)$ and has therefore provided a foundation for the modeling of small-scale turbulence and prediction of turbulent flows in very many contexts, including basic properties of turbulent mean flow profiles (see, for example, Townsend 1976 "The structure of turbulent shear flow" CUP, Launder & Spalding 1972 "Mathematical models of turbulence" Academic Press, Tennekes & Lumley 1972 "A first course in turbulence" MIT Press).

The scale-by-scale energy balance in periodic turbulence is the same as in homogeneous (not necessarily isotropic) turbulence (see Frisch 1995 "Turbulence, the legacy of A.N. Kolmogorov" CUP). In spectral space, this balance is the Lin equation

$$\frac{\partial}{\partial t} E(k,t) = -\frac{\partial}{\partial k}\Pi(k,t) - 2\nu k^2\, E(k,t) \tag{3}$$

where $\Pi(k,t)$ is the interscale energy flux to Fourier modes with wavenumber larger than wavenumber k. As explained in Vassilicos (2015), whereas (2) is compatible with equilibrium turbulence where the inertial range energy flux and dissipation are balanced at all times, the dissipation scaling (1) is not.

This paper's first objective is to show that the new dissipation law (1) also holds at high Reynolds numbers in Direct Numerical Simulations (DNS) of two very different kinds of unsteady periodic turbulence where the interscale balance equation (3) is demonstrably the same as in homogeneous turbulence. The second and in fact most important objective is to demonstrate that at high enough Reynols numbers in these two unsteady turbulent flows, the interscale energy flux $\Pi(k,t)$ at intermediate wavenumbers scales in the same way as the dissipation.

SIMULATIONS AND RESULTS

We conducted spectral DNS of turbulent incompressible fluid flows in a periodic cube. The forcing imposed on the Navier-Stokes equation was $\mathbf{f} = (\sin(2\pi m x/\mathcal{L})\cos(2\pi m y/\mathcal{L}), -\cos(2\pi m x/\mathcal{L})\sin(2\pi m y/\mathcal{L}), 0)$, the very same forcing introduced by Goto, Saito & Kawahara (2015 "Hierarchy of anti-parallel vortex turbes in turbulence at high Reynolds numbers" submitted to J. Fluid Mech.) where $\mathcal{L}$ is the spatial period of the boundary condition and m is an integer. For the decaying turbulence we chose $m = 4$ so that $L(t)$ is sufficiently smaller than $\mathcal{L}$ (i.e. $L(t) < 0.1\mathcal{L}$) during the decay considered, and switched off the force when the dissipation rate $\epsilon(t)$ reached its maximum value. We run five different simulation sizes between 128^3 and 1024^3 for similar resolutions of the smallest eddies, corresponding to five different values of Re_0. To conduct DNS at higher Reynolds numbers, we used $m = 1$ and kept the forcing on throughout. Interestingly, the turbulence driven by this steady force is far from steady and $u'(t)$, $L(t)$ and $\epsilon(t)$ oscillate significantly in time with a time scale of about $20\langle L\rangle/\langle u'\rangle$ (see Goto, Saito & Kawahara 2015). These very low frequency oscillations reflect alternations between turbulence decay periods where $Re_\lambda(t)$ decays and $C_\epsilon(t)$ grows and turbulence build-up periods where $Re_\lambda(t)$ grows and $C_\epsilon(t)$ decays. We run seven different sizes between 64^3 and 2048^3 for similar resolutions, corresponding to seven different values of Re_0 which is now defined as $\langle u'\rangle\langle L\rangle/\nu$. Some of our results are shown in the figure below.

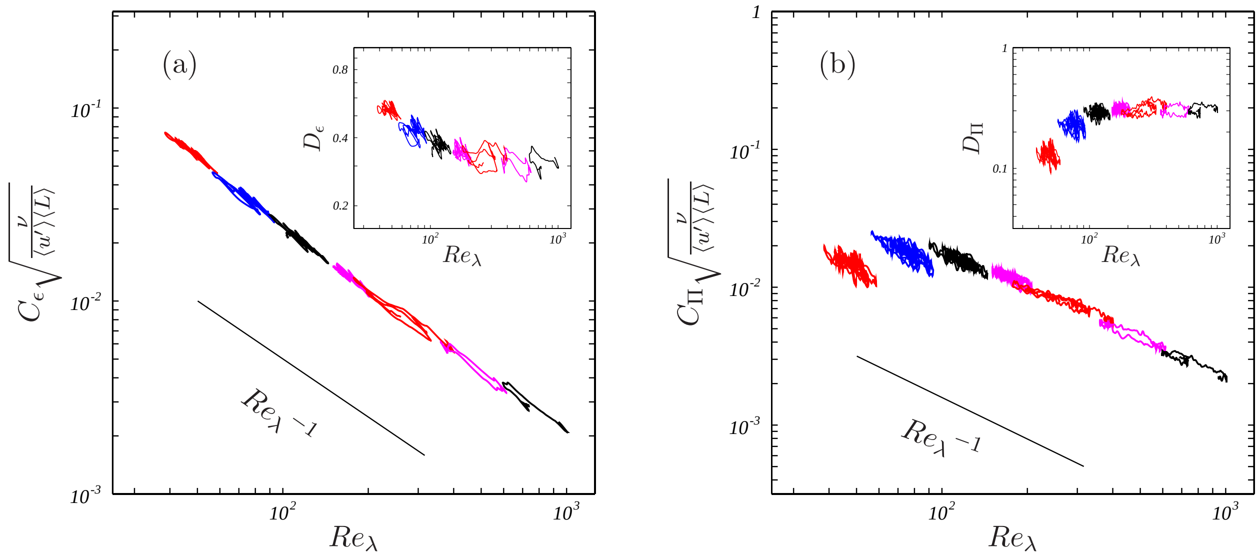

Figure 1. *Left plot:* $C_\epsilon(t)/\sqrt{Re_0}$ (where C_ϵ is defined by (2)) plotted against $Re_\lambda(t)$ for the seven different continuously forced cases in seven different colours showing good collapse on a single continuous line. (Similar results have been obtained in our decaying turbulence for a few turnover times after turning off the forcing.) Insert: $D_\epsilon \equiv C_\epsilon(t)Re_\lambda(t)/\sqrt{Re_0}$ tends to vary around a constant as $Re_\lambda \to \infty$. *Right plot:* $C_\Pi(k,t)$ defined by $\Pi(k,t) = C_\Pi(k,t)u'(t)^3/L(t)$ and plotted for $k = 5k_f$ where $k_f = 2\pi/m$ is the forcing wavenumber (similar results for $k/k_f = 10, 20$ which are also values of k larger than k_f and smaller than $1/\lambda$) against $Re_\lambda(t)$ for the seven different continuously forced cases in the same seven different colours. Collapse on a single continuous line is good for $Re_\lambda > 100$. Insert: $D_\Pi(k,t)$ versus $Re_\lambda(t)$ where $\Pi(k,t) = D_\Pi(k)(\nu Re_0)u'(t)^2/L(t)^2$. (Results similar to both plots have been obtained in our decaying turbulence for a few turnover times after turning off the forcing and will be presented at the conference.)

CONNCLUSIONS

At high enough Reynolds numbers, $\epsilon(t) \sim (\nu Re_0)u'(t)^2/L(t)^2$ in spatially periodic unsteady turbulence, as in turbulence generated by various types of grids in the wind tunnel and in various self-similar axisymmetric turbulent wakes. Re_0 is a Reynolds number defined by inlet/initial/global conditions. At equally high Reynolds numbers in our DNS, $\Pi(k,t) = D_\Pi(k)(\nu Re_0)u'(t)^2/L(t)^2$ for k between k_f and $1/\lambda$. Both our decaying and our forced periodic turbulent flows are such that $\epsilon(t)$ and $\Pi(k,t)$ are not equal. Their scalings characterise non-equilibrium small-scale turbulence universally even though they incorporate dependencies on inlet/initial/global conditions. The equilibrium inertial range balance between $\epsilon(t)$ and $\Pi(k,t)$ and the related $C_\epsilon = Const$ scaling hold together only for turbulence forced so as to keep the energy spectrum time-independent, the ideal situation for the Kolmogorov (1941) theory to apply. Our results motivate the development of a non-equilibrium cascade theory of small-scale turbulence which we also plan to present.