ENERGY TRANSFERS IN SMALL-SCALE AND LARGE-SCALE DYNAMOS

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Abstract We study energy transfers during magnetic energy growth in small-scale and large-scale dynamos. We perform direct numerical simulations for magnetic Prandtl number $\mathrm{Pm} = 20$ and 0.2 in a periodic box on 1024^3 grid. Energy fluxes and shell-to-shell energy transfers indicate that in small-scale dynamo for $\mathrm{Pm} = 20$, the magnetic energy growth takes place due to a non-local energy transfer from large-scale velocity field to small-scale magnetic field. On the other hand, in large-scale dynamo for $\mathrm{Pm} = 0.2$, local energy transfers from large-scale velocity field to large-scale magnetic field takes place.

INTRODUCTION

Generation of magnetic field in a conducting fluid is explained via dynamo theory. The nature of dynamo is highly dependent on a non-dimensional parameter magnetic Prandtl number $\mathrm{Pm}\ (= \nu/\eta)$, where ν is the kinematic viscosity and η is the magnetic diffusivity of the fluid. Typically, it is observed that for $\mathrm{Pm} > 1$ (e.g., Galactic plasma), the magnetic field grows at characteristic length scales smaller than that of the velocity field; this is referred as small-scale dynamo (SSD) [1]. On the other hand, for $\mathrm{Pm} < 1$ (e.g., liquid metal, Earth's core), the magnetic field grows at characteristic length scales larger than that of the velocity field; this is known as large-scale dynamo (LSD). Note however that there are several exceptions to the above categorisation. To understand the growth mechanisms of the magnetic energy in small-scale and large-scale dynamos, we perform magnetohydrodynamic (MHD) simulations for $\mathrm{Pm} = 20$ ($\nu = 0.01$, $\eta = 0.0005$), which yields a small-scale dynamo, and $\mathrm{Pm} = 0.2$ ($\nu = 0.002$, $\eta = 0.01$), which generates a large-scale dynamo.

DYNAMO SIMULATION

We perform direct numerical simulations using a pseudo-spectral code Tarang [2]. The MHD equations are solved in a three-dimensional periodic box of size $(2\pi)^3$ with 1024^3 grid. The time integration is performed using Runge-Kutta fourth-order (RK4) method, and 2/3 rule is used for dealiasing. We apply a nonhelical forcing to the velocity field in a wavenumber band $k = [2, 4]$ such that the kinetic energy supply rate is constant. We first perform a pure fluid simulation with $\nu = 0.01$ (for SSD) and with $\nu = 0.002$ (for LSD) until we obtain steady fluid states. For dynamo simulation, we use the final fluid states and apply a small seed magnetic field (with magnetic energy 10^{-4} unit) in a wave number band $k = [2, 4]$.

To study the energy transfers during the growth of magnetic energy in SSD and LSD, we compute the shell-to-shell energy transfer rates. In MHD turbulence, we discuss three kinds of shell-to-shell energy transfer rates [3, 4]: from velocity to velocity ($U2U$), from magnetic to magnetic ($B2B$), and from velocity to magnetic field ($U2B$). The shell-to-shell energy transfer from the m-th shell of α field to the n-th shell of β field is defined as [3, 4, 5]

$$T^{\beta,\alpha}_{n,m} = \sum_{\mathbf{k}\in n} \sum_{\mathbf{p}\in m} S^{\beta\alpha}(\mathbf{k}|\mathbf{p}|\mathbf{q}), \tag{1}$$

where $S^{\beta\alpha}(\mathbf{k}|\mathbf{p}|\mathbf{q})$ represents energy transfer rate from mode $\mathbf{p}$ of α field to mode $\mathbf{k}$ of β field with the mode $\mathbf{q}$ acting as a mediator. Here the modes of a triad $(\mathbf{k}, \mathbf{p}, \mathbf{q})$ satisfy a condition $\mathbf{k} + \mathbf{p} + \mathbf{q} = 0$. For example, energy transfer rate from $\mathbf{u}(\mathbf{p})$ to $\mathbf{b}(\mathbf{k})$ with $\mathbf{b}(\mathbf{q})$ acting as a mediator is given by

$$S^{bu}(\mathbf{k}|\mathbf{p}|\mathbf{q}) = \Im([\mathbf{k} \cdot \mathbf{b}(\mathbf{q})][\mathbf{b}(\mathbf{k}) \cdot \mathbf{u}(\mathbf{p})]), \tag{2}$$

where $\Im$ denotes the imaginary part of the argument.

For shell-to-shell transfer computations, we divide the wavenumber space into 19 logarithmically binned spherical shells [3, 4, 6]. In Fig. 1, we show shell-to-shell energy transfer rates during the growth of magnetic energy for (a) $\mathrm{Pm} = 20$ (SSD) and (b) $\mathrm{Pm} = 0.2$ (LSD). For SSD, the energy transfers $U2U$ and $B2B$ are forward and local, i.e., the energy transfers occur from small wavenumber shells to large wavenumber shells, and the transfer is predominantly among neighbouring shells (shown in Fig. 1(a)). On the other hand, the $U2B$ transfer is forward and nonlocal, i.e., energy is transferred from small wavenumber (large length scale or forcing scale) u-shells to large wavenumber (small length scale) b-shells. The nonlocal $U2B$ transfer is observed because the velocity modes dominate at small wavenumbers whereas magnetic modes at large wavenumbers. As the dynamo evolves with time, the peak of nonlocal $U2B$ energy transfer shifts towards smaller wavenumbers. In the final stages of the simulation, the magnetic energy grows at comparatively smaller wavenumbers [6].

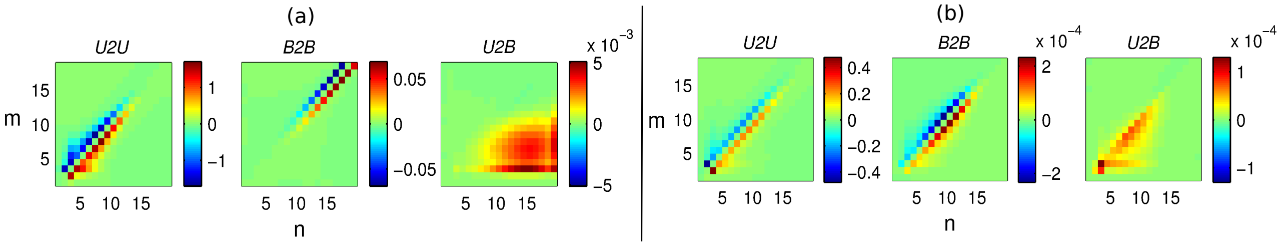

Figure 1. Shell-to-shell energy transfer rates: velocity to velocity field ($U2U$), magnetic to magnetic ($B2B$), and velocity to magnetic field ($U2B$) for (a) small-scale dynamo with $\mathrm{Pm} = 20$ (adopted from [6]) and (b) large-scale dynamo with $\mathrm{Pm} = 0.2$ (adopted from [7]). Here horizontal axes represent the receiver shells, while vertical axes represent the giver shells. The energy transfers $U2U$ and $B2B$ are forward and local in both the cases, whereas the $U2B$ transfer is nonlocal in small-scale dynamo, but local in large-scale dynamo.

The shell-to-shell energy transfer rates in LSD simulation for $\mathrm{Pm} = 0.2$ is depicted in Fig. 1(b). The energy transfers $U2U$ and $B2B$ are forward and local, similar to what is observed in SSD for $\mathrm{Pm} = 20$. The $U2B$ transfer is also forward and local unlike SSD. The peak of $U2B$ transfer is concentrated at small wavenumbers and it does not shift with time. In other words, the growth of magnetic energy takes place mainly due to local energy transfers from large-scale velocity field to large-scale magnetic field because both the velocity and magnetic modes are significant at small wavenumbers.

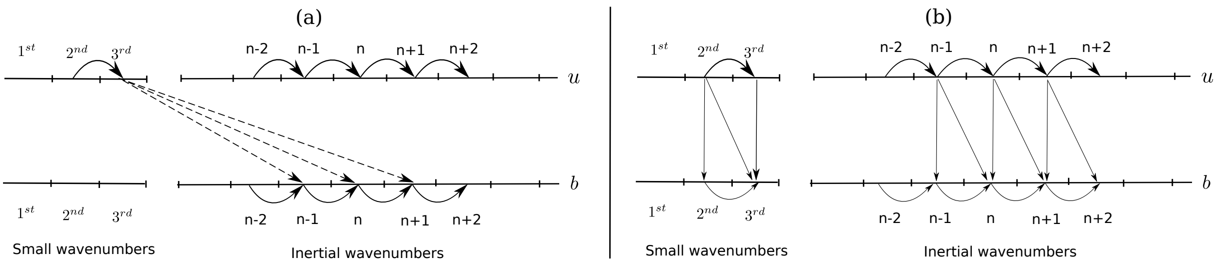

Figure 2. Schematic diagrams of shell-to-shell energy transfers among u-shells and b-shells for (a) small-scale dynamo with $\mathrm{Pm} = 20$ and (b) large-scale dynamo with $\mathrm{Pm} = 0.2$ (adopted from [7]).

In Fig. 2, we exhibit schematic diagrams of shell-to-shell energy transfers for (a) $\mathrm{Pm} = 20$ (SSD) and (b) $\mathrm{Pm} = 0.2$ (LSD). For SSD, $U2U$ and $B2B$ energy transfers are forward and local, whereas $U2B$ transfer is forward and nonlocal (see dashed lines in Fig. 2(a)). For LSD, all the three energy transfers: $U2U$, $B2B$, and $U2B$ are forward and local (see Fig. 2(b)).

In conclusion, the energy transfer studies show that in small-scale dynamo, the growth of magnetic energy takes place due to a nonlocal energy transfer from large-scale velocity field to small-scale magnetic field whereas in large-scale dynamo, the magnetic energy grows due to local energy transfers between velocity and magnetic fields.

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