WAVENUMBER-FREQUENCY SPECTRA IN THE LOGARITHMIC LAYER OF WALL TURBULENCE

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Abstract We study space-time correlations of wall-bounded turbulence in terms of wavenumber-frequency spectra of the streamwise velocity component. The spectra are obtained from Large Eddy Simulations (LES), which provide a full space-time record of the flow. We find that the frequency distributions exhibit a Doppler shift, which is a consequence of mean flow advection, as well as a considerable Doppler broadening, consistent with the Kraichnan-Tennekes random sweeping hypothesis. For wall-bounded turbulence, both of these effects vary with the wall distance and are closely related to the logarithmic behavior of the mean velocity profile and the velocity fluctuation profiles. We incorporate these observations into a simple analytical model for the wavenumber-frequency spectrum based on an advection equation featuring advection of the small-scale velocity fluctuations with a mean and a large-scale random-sweeping velocity. The model is found to be in very good agreement with the LES data. Potential applications of the model spectrum, e.g., to quantify the spatio-temporal structure of fluctuations in wind energy conversion, will be discussed.

SPACE-TIME CORRELATIONS OF TURBULENT WALL-BOUNDED FLOWS

Turbulent wall-bounded flows exhibit a rich spatio-temporal structure (see, e.g., [3, 1] for recent review articles), which can be appreciated from space-time plots of the streamwise velocity fluctuations (see fig. 1). As can be seen from these plots, turbulent fluctuations are predominantly advected in the streamwise direction with the mean velocity, consistent with Taylor's frozen eddy hypothesis. Furthermore, small-scale eddies are swept by larger-scale ones which contributes to a perturbation of the space-time pattern. On the statistical level, one of the most fundamental quantities capturing the space-time correlations of such a flow is the wavenumber-frequency spectrum, which is at the center of this study.

To study this spectrum, we have performed LES on a domain of $L_x/H \times L_y/H \times L_z/H = 4\pi \times 2\pi \times 1$, discretized on a grid with $1024 \times 512 \times 256$ grid points and collected data in the statistically stationary state spanning 8.2×10^4 time steps. The wavenumber-frequency spectrum, projected to the k_1-ω plane, is shown in the left panel of fig. 2. The most striking features of this spectrum are a Doppler shift of frequencies as well as a Doppler broadening, which we seek to capture in a simple model.

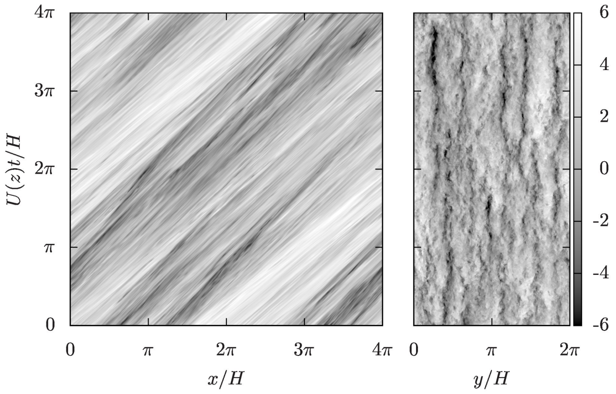

Figure 1. Space-time plots of the streamwise velocity fluctuations in a turbulent wall-bounded flow at height $z/H \approx 0.154$ from LES. Left: streamwise cut, right: spanwise cut. The color bar is given in units of the friction velocity. Figure taken from [6].

SPACE-TIME SPECTRA FROM RANDOM SWEEPING HYPOTHESIS WITH MEAN FLOW

The model is based on the Tennekes-Kraichnan random sweeping hypothesis with additional mean flow [2, 4]. In this model, the small-scale velocity fluctuations are considered as passively advected (in planes at a fixed height z) by the mean velocity as well as the large-scale random sweeping velocity. By a scale-separation argument, the random sweeping velocity is taken constant in space and time, and for simplicity, with a Gaussian ensemble distribution. Under such drastically simplified conditions a soluble model is obtained [5, 6], leading to the prediction that the wavenumber-frequency spectrum $E_{11}(\boldsymbol{k}, \omega; z)$ can be written as the product of the wavenumber spectrum $E_{11}(\boldsymbol{k}; z)$ and a Gaussian frequency

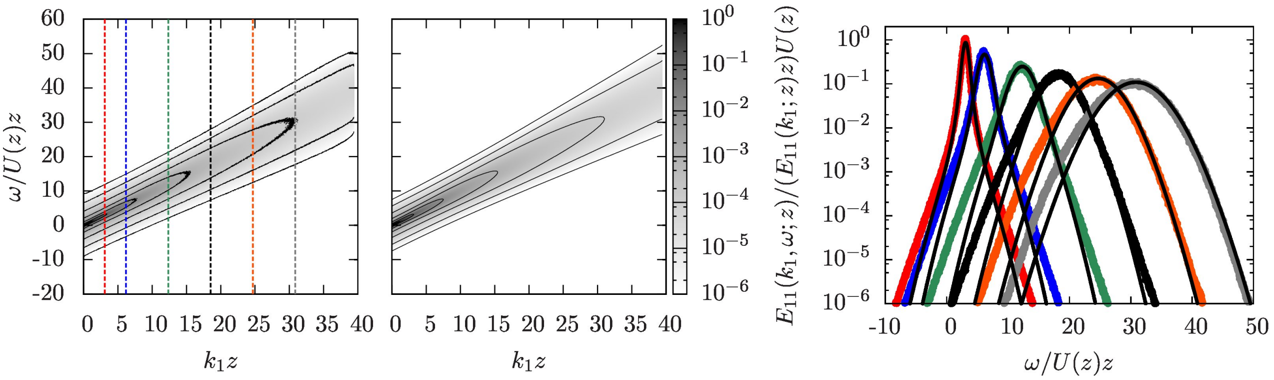

Figure 2. k_1-ω spectra of the streamwise velocity component at $z/H \approx 0.154$. In the left panel the spectrum is evaluated directly from LES. The middle panel shows the spectrum obtained from the random sweeping hypothesis with mean flow, for which only the wavenumber spectrum and the mean and random-sweeping velocities evaluated from the LES data have been used. The right panel shows normalized cuts through the k_1-ω spectrum from LES (colors) together with the results from the linear random advection model (black lines). The positions of the cuts are indicated in the left panel. Figure taken from [6].

distribution:

$$E_{11}(\boldsymbol{k},\omega;z) = E_{11}(\boldsymbol{k};z)\left[2\pi\left\langle(\boldsymbol{v}\cdot\boldsymbol{k})^2\right\rangle\right]^{-1/2}\exp\left[-\frac{(\omega-\boldsymbol{k}\cdot\boldsymbol{U})^2}{2\left\langle(\boldsymbol{v}\cdot\boldsymbol{k})^2\right\rangle}\right]. \tag{1}$$

Here, $\boldsymbol{k} = (k_1, k_2)^{\mathrm{T}}$ denotes the wavevector in horizontal planes, ω is the frequency, $\boldsymbol{U}$ is the mean velocity and $\boldsymbol{v}$ the large-scale random sweeping velocity. As can be expected on physical grounds, mean flow advection leads to a Doppler shift of frequencies, whereas random sweeping leads to a Doppler broadening.

To test this prediction we compare the wavenumber-frequency spectrum evaluated according to the right-hand side of eq. (1) to the one directly obtained from LES. The middle panel shows the prediction (1), for which only the wavenumber spectrum as well as the mean velocity and variance of the sweeping velocity have been estimated from the LES data. For a quantitative comparison, normalized cuts are shown in the right panel. Given the underlying simplifying assumptions of the model, a remarkable agreement is found.

ANALYTICAL MODEL PARAMETERIZATION

While the random advection model with mean flow leads to the prediction (1) as a main result, it does neither determine the wavenumber spectrum, nor does it specify the height dependence of the mean and random sweeping velocities. By modeling these quantities explicitly and combining them with eq. (1), an analytical model is obtained which may turn out to be useful for future purposes (e.g. to characterize velocity fluctuations in wind farms). In short, the wavenumber spectrum is constructed from a small-scale contribution for which we assume small-scale isotropy as well as a large-scale contribution for which we try to incorporate features typically observed on the large scales of wall-bounded turbulent flows. To model the height-dependence of Doppler shift and Doppler broadening, we consider the range in which the mean and the random sweeping variance exhibit a logarithmic dependence, such that we can parameterize these effects with log laws. The model is described in more detail and benchmarked in [6].

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