A DYNAMIC SUBFILTER-SCALE STRESS MODEL FOR LARGE EDDY SIMULATIONS BASED ON PHYSICAL FLOW SCALES

Amirreza Rouhi[1], Ugo Piomelli[1] & Bernard J. Geurts[2]
[1]*Queen's University, Kingston, Ontario, Canada*
[2]*University of Twente, Enschede, The Netherlands*

Abstract We propose a new definition of the length scale in an eddy-viscosity model for large-eddy simulations (LES). This formulation extends and generalizes a previous proposal [Piomelli, Rouhi and Geurts, *Proc. ETMM10*, 2014], in which the LES length scale was expressed in terms of the integral length-scale of turbulence determined by the flow characteristics and explicitly decoupled from the simulation grid; this approach was named Integral Length-Scale Approximation (ILSA). As in the original ILSA, the model coefficient was determined by the user, and required to maintain a desired contribution of the unresolved, subfilter scales (SFS) to the global transport. We propose a local formulation (local ILSA) in which the model coefficient is local in space, allowing a precise control over SFS activity as a function of location. This new formulation preserves the properties of the global model; application to channel flow and backward-facing step verifies its features and accuracy.

Keywords. turbulence modelling, large eddy simulation, filtering.

FORMULATION

Large Eddy Simulation (LES) is based on resolving the large, energy-carrying, eddies and modelling the small, unresolved, ones. Filtering is the mathematical operation performed to separate the large eddies from the small, subfilter, scales (SFS); the filter-width Δ defines the physical size of the smallest retained eddies, and is linked to the model length-scale. If the large scales are expected to account for most of the momentum transport, Δ must be a fraction of the integral scale L. In current-day LES practice, however, Δ is generally determined based on the grid size h, *i.e.*, $\Delta \propto h$. This choice decouples the filter-width from its physical meaning, leading to some undesirable characteristics of the model for the unresolved scales . Piomelli *et al.*[4] proposed to resolve these deficiencies by associating the filter-width to the flow properties, linking Δ to an approximation of the integral length-scale computed using the resolved turbulent kinetic energy, $\mathcal{K}_{\rm res} = 1/2\overline{u}_i'\overline{u}_i'$ and the dissipation rate, $\epsilon_{\rm tot} = 2\left(\nu + \nu_{\rm sfs}\right)\overline{s}_{ij}'\overline{s}_{ij}'$, where $\overline{s}_{ij}'$ is the fluctuating part of the strain-rate tensor. The Integral Length-Scale Approximation (ILSA) model was proposed, in which the integral length scale is estimated as $L_{\rm est} = \mathcal{K}_{\rm res}^{3/2}/\epsilon_{\rm tot}$, and the model length scale is proportional to L, resulting in an eddy-viscosity approximation of the form:

$$\nu_{\rm sfs} = \left(C_m\Delta\right)^2\left|\overline{S}\right| = C_k^2\frac{\left\langle\mathcal{K}_{\rm res}\right\rangle^3}{\left\langle\epsilon_{\rm tot}\right\rangle^2}\left|\overline{S}\right| \tag{1}$$

In (1), C_k is the only model parameter. In the original ILSA model [4] C_k was determined by performing a number of auxiliary coarse-grid simulations in the configuration of interest, and finding the value of C_k that yields a desired value of the global contribution of subfilter eddies to the total dissipation rate (Meyers *et al.*, [3]):

$$s_\epsilon^{\mathcal{V}} = \frac{\langle\epsilon_{\rm sfs}\rangle_{\mathcal{V},t}}{\langle\epsilon_{\rm tot}\rangle_{\mathcal{V},t}} = \frac{\left\langle 2\nu_{\rm sfs}\overline{S}_{ij}\overline{S}_{ij}\right\rangle_{\mathcal{V},t}}{\left\langle 2\left(\nu_{\rm sfs}+\nu\right)\overline{S}_{ij}\overline{S}_{ij}\right\rangle_{\mathcal{V},t}} = \frac{\text{subfilter dissipation rate}}{\text{total dissipation rate}} \tag{2}$$

Since the quantities in (2) were averaged over the computational volume and time ($\langle ... \rangle_{\mathcal{V},t}$), C_k is constant over the domain; locally, the contribution of the unresolved scales to the dissipation could differ from the desired level of $s_\epsilon^{\mathcal{V}}$. In this study we propose a local formulation for ILSA, that allows a more precise assignment of the contribution of the unresolved scales to the transport and extends the range of applicability of the model to more complex problems.
First, since $s_\epsilon^{\mathcal{V}}$ approaches unity as the Reynolds number is increased, we introduce a more general measure of SFS activity based on the contribution of unresolved eddies to the Reynolds stresses.

$$s_\tau^\Omega = \left[\frac{\left\langle\tau_{ij}^a\tau_{ij}^a\right\rangle_\Omega}{\left\langle\left(\tau_{mn}^a + R_{mn}^a\right)\left(\tau_{mn}^a + R_{mn}^a\right)\right\rangle_\Omega}\right]^{1/2} \tag{3}$$

$$\tau_{ij}^a = \tau_{ij} - \frac{\delta_{ij}}{3}\tau_{kk} = -2\nu_{\rm sfs}\overline{S}_{ij}, \quad R_{ij}^a = \overline{u}_i'\overline{u}_j' - \frac{\delta_{ij}}{3}\overline{u}_k'\overline{u}_k' \tag{4}$$

where τ_{ij}^a and R_{ij}^a are the anisotropic parts of the modelled and resolved Reynolds stresses respectively. Note that $\overline{u}_i'$, the fluctuating part of the velocity field, is known from the simulation results. In (3), the averaging $\langle ... \rangle_\Omega$ is performed only over time and (if appropriate) in directions in which the flow is homogeneous. This implies that C_k is a function of space in inhomogeneous flows, and the desired subfilter activity can be achieved, potentially, at each point. Once s_τ^Ω is assigned, substituting (1) and (4) into (3) and rearranging terms yields a quadratic equation for C_k that can be solved each time step and in each grid cell.

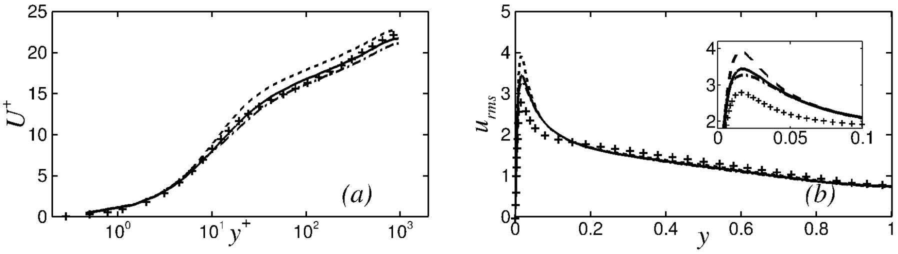

Figure 1. Channel flow, $Re_\tau = 1,000$, $64 \times 97 \times 64$ grid points. Mean velocity (a) and u_{rms} velocity (b). ——— Local ILSA with $s_\tau^\Omega = 0.022$; -·- global ILSA model [4] with $s_\tau^V = 0.022$; - - - dynamic model [1]; + DNS [2].

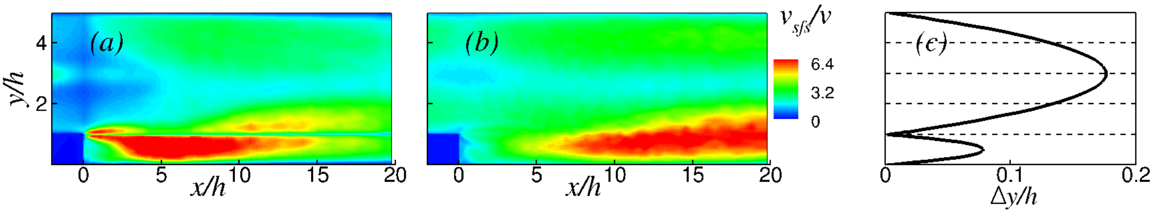

Figure 2. Ratio of the eddy viscosity ($\nu_{\rm sfs}$) to the kinematic viscosity for the backward-facing step on the coarse grid using *(a)* the dynamic model and *(b)* the local ILSA model with $s_\tau^\Omega = 0.022$. *(c)* grid size across the domain ($1.3 \times 10^{-3} \le \Delta y/h \le 0.18$).

RESULTS

The local ILSA model was used in channel flow at $Re_\tau = u_\tau \delta/\nu = 1,000$ and $2,000$ (where δ is the channel half-width, u_τ the friction velocity and ν the kinematic viscosity). Figure 1 shows the comparison between the local ILSA model (in a case with $s_\tau^\Omega = 0.022$), the global ILSA model [4] (using a constant C_k that yields $s_\tau^V = 0.022$ averaged over time and volume) and the dynamic model [1]. As was the case for the global ILSA model, the local ILSA is also more accurate than the dynamic model on a coarse grid; relating the model length scale to $L_{\rm est}$ produces a large eddy viscosity in the buffer layer that compensates for the lack of momentum transport due to the under-resolution of the near-wall eddies. As the grid is refined, these eddies are resolved better, and the eddy viscosity decreases to a grid-independent level controlled only by the flow conditions and the value assigned externally to s_τ^Ω. This is a desirable practice in LES [5]. Using a low value of s_τ^Ω requires a finer grid resolution to reach grid convergence, while higher values require lower computational effort, but increase the contribution of the model to the transport (and, thus, may increase the modelling errors). Sensitivity tests indicated that the model is accurate for $s_\tau^\Omega \le 0.03$.

The local ILSA model was also used in calculations of a backward-facing step at $Re_c = 28,000$ (based on the centerline velocity at the inlet and step height). The set up followed the experiment by Vogel & Eaton [7]. Three levels of grid resolution were tested; coarse ($256 \times 100 \times 64$), intermediate ($384 \times 150 \times 96$) and fine ($512 \times 200 \times 128$). Comparing the proposed model with $s_\tau^\Omega = 0.022$ to the dynamic model confirmed that the local ILSA model has better accuracy on the coarse grids; the mean velocity and Reynolds stresses (not shown) where in good agreement with the experimental data when $s_\tau^\Omega \le 0.03$.

Figure 2 compares the eddy viscosity obtained with the two models. Since the grid is refined in the shear layer emanating from the step corner (Figure 2*(c)*), the dynamic model yields very low eddy viscosity in this region, while the local ILSA model gives a more uniform distribution of the eddy viscosity (Figures 2*(a)* and *(b)*). This distribution is less prone to the modelling and aliasing errors that appear when sudden grid discontinuities occur [6].

References

[1] M. Germano, U. Piomelli, P. Moin, and W. H. Cabot. A dynamic subgrid-scale eddy viscosity model. *Physics of Fluids*, **3**(7):1760–1765, 1991.

[2] S. Hoyas and J. Jiménez. Scaling of the velocity fluctuations in turbulent channels up to reτ= 2003. *Physics of Fluids*, **18**(1):011702, 2006.

[3] J. Meyers, B. J. Geurts, and M. Baelmans. Database analysis of errors in large-eddy simulation. *Physics of Fluids*, **15**(9):2740–2755, 2003.

[4] U. Piomelli, A. Rouhi, and B. J. Geurts. A grid-independent length scale for large-eddy simulations. *Journal of Fluid Mechanics*, (submitted).

[5] S. B. Pope. Ten questions concerning the large-eddy simulation of turbulent flows. *New journal of Physics*, **6**(1):35, 2004.

[6] M. Vanella, U. Piomelli, and E. Balaras. Effect of grid discontinuities on large-eddy simulation statistics and flow fields. *Journal of Turbulence*, (9), 2008.

[7] J. C. Vogel and J. K. Eaton. Combined heat transfer and fluid dynamic measurements downstream of a backward-facing step. *Journal of heat transfer*, **107**(4):922–929, 1985.