NUMERICAL STUDY ON EFFECT OF REYNOLDS NUMBER ON DYNAMO ACTION

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Abstract We study the kinematic dynamo problem of a two dimensional turbulent flow with the third velocity component being advected as a passive scalar (2.5D flow). Both helical and nonhelical forcing is considered. The low-dimensionality of the system allows us to study it for a wide range of parameters of the system, here specifically the Reynolds number and the magnetic Reynolds number. We show that the small scale dynamo action depends on the Reynolds number. The critical magnetic Reynolds number after which small magnetic perturbations starts to grow for the nonhelical forcing case is found to be independent of the Reynolds number.

INTRODUCTION

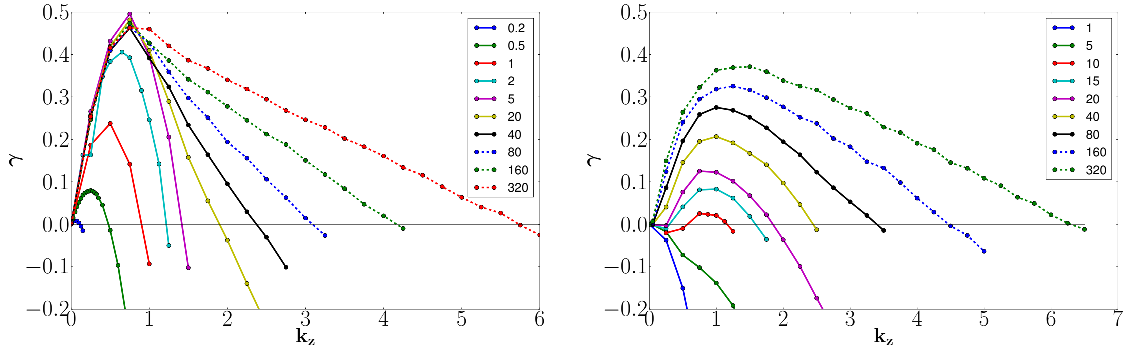
Dynamo action in $2.5D$ flows has been studied by Robert [6] where he considered a periodic array of vortices in which a small perturbation in the magnetic field was amplified by the alpha effect. Since then there have been many studies on the alpha effect in different flow fields. Alpha effect is directly correlated with the helicity in the flow and in general dies down at infinite Reynolds number. The growth of magnetic fields at smaller scales happens through the stretch-twist effect of the small scale dynamo [7]. The small scale dynamo is predominant in flows which are chaotic or turbulent leading to a dynamo effect in the limit of infinite Reynolds number. For the case of nonhelical dynamos there is a critical magnetic Reynolds number below which small magnetic perturbations dies out and beyond which growth of magnetic fields are possible. Recent work has been to study how this critical magnetic Reynolds number changes with the Reynolds number of the flow. Recent results by [1–5] show that there exists a critical Rm in the limit of infinite Re which corresponds to the limit of $Pr \to 0$, Pr being the Prandtl number defined as $Pr = \frac{Rm}{Re}$. Compared to the three dimensional flow, the $2.5D$ system is computationally more tractable and allows us to do a detailed parametric study at two limits $Pr \to \infty$ and $Pr \to 0$.

RESULTS

The $2.5D$ flow configuration corresponds to a rotating three-dimensional flow at high rotation speed since in fast rotating flows the variations along the axis of rotation is suppressed due to the Coriolis force. The magnetic field is considered to be infinitesimal and its effect on the velocity field is neglected. Since we neglect the Lorentz force the governing equation for the magnetic field becomes linear with respect to the magnetic field. Due to the invariance along the third direction we consider magnetic field perturbation of the form $\mathbf{B} = \mathbf{B}(x, y)e^{i\,k_z\,z}$ where x, y denote the spatial coordinates in-plane and z denotes the coordinate perpendicular to the plane. The wavenumber k_z is one of the parameters in this study. The other nondimensional numbers are the Reynolds number $Re_f = f_0^{1/2}/\left(k_f^{1/2}\nu\right)$ and the magnetic Reynolds number $Rm_f = f_0^{1/2}/\left(k_f^{1/2}\mu\right)$ and those from the hypo dissipation which are kept constant at a value where no large scale condensate is formed. Thus we have four parameters to be varied which are, $k_f L, k_z L, Re_f, Rm_f$. Two different forcing, one with mean helicity and the other without mean helicity is studied. We calculate the growth rate of the magnetic field $\gamma\left(k_z, Re_f, Rm_f\right) = \lim_{t\to\infty} \frac{1}{2t} log \frac{B^2(t)}{B^2(0)}$. Figure 1 shows γ as a function of k_z for different values of Rm_f for a fixed Re_f for helical and nonhelical forcing. For each Rm_f we have a region of k_z where growth of magnetic field is possible. In the case of the helical forcing, for very small k_z growth is due to the alpha effect while for large k_z we have the small scale dynamo effect. For the nonhelical forcing there is no alpha dynamo effect hence there is no dynamo at small Rm_f and small k_z. As we increase the Rm_f we see the growth rate increasing and for large k_z, Rm_f values the behavior is similar to the helical forcing case. There is a critical Rm_f below which magnetic perturbation dies down for any k_z. This value is found to be around $Rm_c \approx 10$ and is independent of Re_f for large Re_f. For both forcing considered here there exist a cut off wavenumber k_z^c above which there is no dynamo effect, here $k_z^c = f\left(Re_f, Rm_f\right)$. Figure 2a shows the cut-off k_z^c as a function of Rm_f for different Re_f for the helical case. Similar behavior is observed for the nonhelical forcing. Figure 2b shows the saturation of the maximum growth rate γ_{max} over all values of k_z as a function of Rm_f for different Re_f for the helical forcing case.

DISCUSSION

The helical forcing case shows that alpha effect dominates the dynamo growth rates at small Pr limit. For the limit $Pr \gg 1$ the small scale dynamo sets in and cut-off wavelength k_z is found to strictly depend on Re. The dependence



(a) (b)

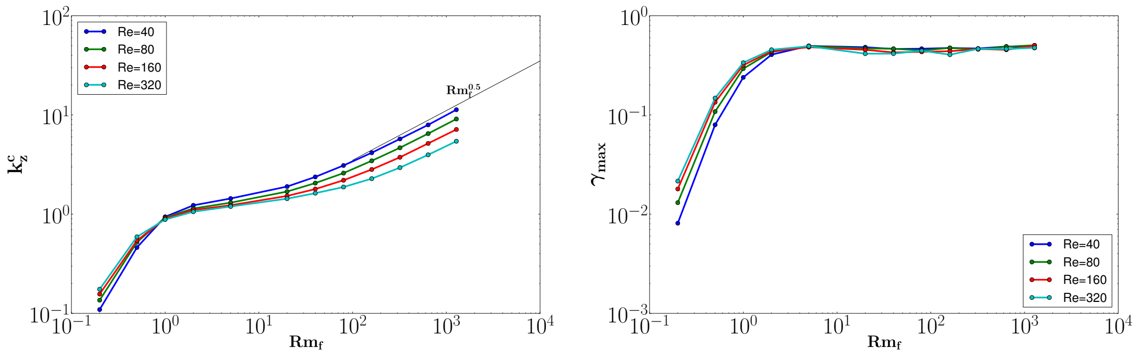
Figure 1. Plot of growth rate of the magnetic field γ as a function of k_z for (a) helical , (b) nonhelical, forcing cases.

of the cut-off wavelength is found to scale as $Rm_f^{1/2}$ in the limit of $Pr \gg 1$. A scaling argument that could explain this behavior would be to balance the viscous term with the source term in the induction equation.

$$\mu \Delta b^2 \sim u \nabla b^2 \tag{1}$$

$$\mu k_z^2 b^2 \sim u b^2/\ell \implies k_z \sim k_f \sqrt{Rm_f} \tag{2}$$

Here the large scale magnetic fields are predominantly dissipated by the modulation k_z. In the case of the nonhelical forcing there exists a critical magnetic Reynolds number above which dynamo effect is possible. In the limit of $Pr \ll 1$ at large Re we find that the $Rm_c \approx 10$ which implies that the onset of dynamo action does not depend on the small scales of turbulent motion. In the limit of large Rm_f the behavior of both the types of forcing tends to be the same. In future we will study the dependencies of the observed scaling as we change the forcing wavenumber k_f.



(a) (b)

Figure 2. (a) Plot of the cut off wavenumber k_z^c as a function of Rm_f for different values of Re_f, (b) Plot of maximum growth rate γ_{max} as a function of Rm_f for different Re_f, both plots for helical forcing.

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