THE EFFECT OF TEMPERATURE FLUCTUATIONS ON THE SPREAD OF A BUOYANT PLUME

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Abstract Emissions from many natural and anthropogenic sources are hot compared with the surrounding ambient air. Such buoyancy effects cause the emitted plume to rise, increasing the effective source height and significantly decreasing the maximum ground level concentrations (in the vicinity of the source). A major aspect that distinguishes buoyant and passive dispersion is that buoyant fluid particles create their own turbulence and hence exchange processes between the plume and its environment need to be accounted for. The inclusion of plume rise in Lagrangian stochastic models (LSMs) of turbulent dispersion has been considered by many authors but the interaction of the buoyant plume with the environment (by means of entrainment) is difficult to model in a Lagrangian framework. Webster and Thomson [8] formulated a hybrid model in which the mean flow is calculated from a simple plume model and the fluctuations of velocity are calculated using an LSM. They model the effect of turbulence generated by the plume by an additional random increment to the position of a particle. Here, instead of including this extra term, we add a stochastic differential equation (SDE) for the temperature fluctuations suitably coupled with the SDE for the velocity fluctuations. The interaction of temperature and velocity fluctuations, directly related to the turbulence within the plume, determines the plume's spread. The results of the model are compared with large-eddy simulation (LES) of buoyant plumes in a uniform crosswind and also with the plume generated by the explosion and fire at the Buncefield oil depot in 2005 using realistic profiles of the wind speed and direction and thermal stratification.

A HYBRID MODEL FOR BUOYANT PLUME RISE

The equations governing the rise of a buoyant plume in a uniform crossflow are are well known (e.g. [1]): they describe the evolution of the volume, momentum and buoyancy fluxes and are collectively known as the plume equations. Several authors [4, 5] have attempted to model buoyant plume rise using a Lagrangian approach. Here we use as a starting point a hybrid model [8] in which the mean flow is calculated from a simple plume model and the velocity fluctuations are calculated using a Lagrangian stochastic model (LSM) that satisfies the well-mixed condition [7]. Webster and Thomson [8] only considered fluctuations in the velocity and not the temperature; we consider both fluctuations of the velocity and temperature. Whereas the effect of turbulence generated by the plume is modelled by [8] with an additional random increment to the position of a fluid particle, we allow the interaction of temperature and velocity fluctuations to generate the observed spread of the buoyant plume. Moreover we construct parameterisations of the turbulent time-scale and dissipation rate (which are required by the model) from the plume turbulence rather than the ambient turbulence as was done by [8].

The plume equations for the mean vertical velocity, $\overline{w}$, mean potential temperature, $\overline{\theta}$ and the plume's radius, b, are used as a starting point for constructing stochastic differential equations (SDEs) for the fluctuating vertical velocity, w', and potential temperature, θ'. The SDE for w' takes the form

$$\mathrm{d}w' = \frac{g\theta'}{\theta_0}\,\mathrm{d}t - E\frac{w'}{b^2}\,\mathrm{d}t - \frac{w'}{T_L}\mathrm{d}t + \frac{1}{2}\left(\frac{1}{w} + \frac{w'}{\sigma_w^2}\right)\mathrm{d}\sigma_w^2 + \sqrt{C_0\varepsilon}\,\mathrm{d}W \tag{1}$$

where $w = \overline{w} + w'$, T_L is the time scale on which w' changes, σ_w^2 is the vertical-velocity variance, ε is the mean kinetic energy dissipation rate and C_0 is the constant of proportionality in the second-order Lagrangian velocity structure function which typically has a value in the range $5-7$ in homogeneous isotropic turbulence (we choose $C_0 = 5$). In (1) g is the acceleration due to gravity, θ_0 is a reference potential temperature and E is the entrainment rate. The SDE for θ' is given by

$$\mathrm{d}\theta' = -E\frac{\theta'}{b^2}\mathrm{d}t - \frac{\theta'}{T_\theta}\mathrm{d}t - \frac{w'}{w}\mathrm{d}\overline{\theta} + \sqrt{C_\theta\varepsilon_\theta}\,\mathrm{d}W_\theta \tag{2}$$

where T_θ is the time scale on which θ decorrelates, ε_θ is the mean scalar dissipation rate and C_θ is the Obukhov-Corrsin constant which typically has a value of 1.6 [6]. The form of (2) is similar to that considered by [3]. For simplicity we assume that the turbulent temperature statistics are homogeneous. The initial values of w' and θ' are drawn from a joint Gaussian distribution with zero mean and variances σ_w^2 and σ_θ^2. We choose $\sigma_w = \alpha|\overline{w}|$, where α is an entrainment constant, and $T_L = b/|\overline{w}|$. Since

$$T_L = \frac{2\sigma_w^2}{C_0\varepsilon},$$

it follows that

$$\varepsilon = \frac{2\alpha^2\overline{w}^3}{C_0 b}.$$

The mean scalar dissipation rate is given by

$$\varepsilon_\theta = \frac{2\sigma_\theta^2}{C_\theta T_\theta}$$

where T_θ is chosen to be equal to T_L. We specify $\sigma_\theta = \gamma|\overline{\theta} - \theta_a|$ in which θ_a is the ambient potential temperature and γ is a tunable constant.

COMPARISON WITH LES

The model is first compared with LES of a buoyant plume in a uniform crosswind with constant buoyancy frequency [2]. The value of γ is estimated by comparing the LES profiles of σ_θ with $|\overline{\theta} - \theta_a|$ along the centreline of the plume. We find that the best fit is given with γ in the range $0.1 \lesssim \gamma \lesssim 0.5$ and that there is little systematic variation with the non-dimensional crosswind velocity, $\tilde{U}$. Comparison of the scalar concentration with the LES equivalent shows generally good agreement for a range of $\tilde{U}$ values except when $\tilde{U}$ becomes very small. In general, the height of the peak concentration is similar to that of the LES plume and the best results for the spread of the plume are achieved when $\gamma = 0.5$.

As a further test and illustration of the model, we consider the explosion and fire at the Buncefield oil depot in December 2005 which produced the largest plume of black carbon in Europe since the end of the second world war. Comparisons of LES of this plume with an LSM of the form proposed by [8], i.e. with no temperature fluctuations, showed that the LES results had a greater vertical spread than the LSM results (see Fig. 18 of [2]). This observation has, in part, motivated the present study. The model is extended to allow for non-uniform profiles of wind speed and direction and thermal stratification. Figure 1 shows the scalar concentration computed from the LSM with and without temperature fluctuations θ'. Results are presented with $\gamma = 0.25$ and $\gamma = 0.5$. The LES results are also shown in the same figure. It can be seen that the model plumes generally compares well with the LES plume and that the spread of the model plumes increases with increasing γ (as expected). Overall, the best results are obtained with $\gamma = 0.5$.

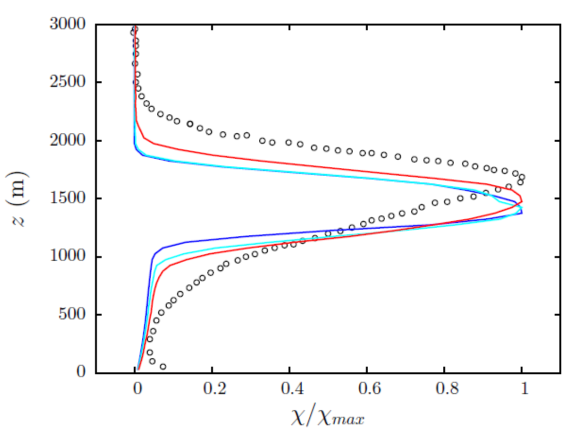

Figure 1. The scalar concentration χ normalised by its maximum value for the Buncefield case described in the text: the black circles are the LES results; the model plumes with $\theta' \neq 0$ are shown by the red and cyan lines for $\gamma = 0.5$ and $\gamma = 0.25$ respectively; the blue line is the model plume with $\theta' = 0$.

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