GEOSTROPHIC CONVECTIVE TURBULENCE: THE EFFECT OF BOUNDARY LAYERS

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Abstract We conduct computations of rotating Rayleigh–Bénard convection in the so-called geostrophic regime, characterized by strong thermal forcing (high Rayleigh numbers) and strong rotation (small Ekman numbers). We employ the full Navier–Stokes equations in our computations and compare no-slip and stress-free boundaries for the plates. The Ekman boundary layers, that exist in the no-slip case but not for stress-free, enhance convective heat transfer and prevent the formation of large-scale flow structures.

GEOSTROPHIC CONVECTION

Rotating Rayleigh–Bénard convection (RRBC) [7] is the flow between two horizontal corotating parallel plates, driven by heating of the bottom plate and cooling of the top plate. It is a simple model system for many geophysical and astrophysical flow settings, like the atmosphere and oceans. The strength of the buoyant forcing is expressed as the Rayleigh number $Ra = g\beta\Delta L^3/(\nu\kappa)$, with g the gravitational acceleration, β the thermal expansion coefficient of the fluid, Δ the applied temperature difference between the plates, L their separation, and ν and κ the kinematic viscosity and thermal diffusivity of the fluid, respectively. The Prandtl number $Pr = \nu/\kappa$ characterizes the fluid. The Ekman number $Ek = \nu/(2\Omega L^2)$ quantifies the relative importance of rotation; Ω is the angular velocity.

A characteristic feature of geophysical flows is the combination of very large Ra (up to 10^{20}) and very small Ek (as small as 10^{-15}). Conventional experiments and DNS studies typically attain values of $Ra \lesssim 10^{10}$ and $Ek \gtrsim 10^{-6}$ [7]. Recent computations using a reduced set of governing equations in the limit of rapid rotation [3, 4] have revealed that these numbers are right around the transition to the so-called *geostrophic* regime of turbulent convection: a state of 'featureless' turbulence, without coherent vortical columns [7] as expected from the Taylor–Proudman constraint.

The original studies [3, 4] employed stress-free plates. Here we extend this work by employing DNS of the full Navier–Stokes equations in the geostrophic regime of RRBC and by comparing no-slip and stress-free plates. We employ a horizontally periodic box and apply a constant $Pr = 1$.

HEAT TRANSFER

The convective heat transfer, expressed as the Nusselt number Nu, is calculated for all runs and depicted in figure 1(a). A

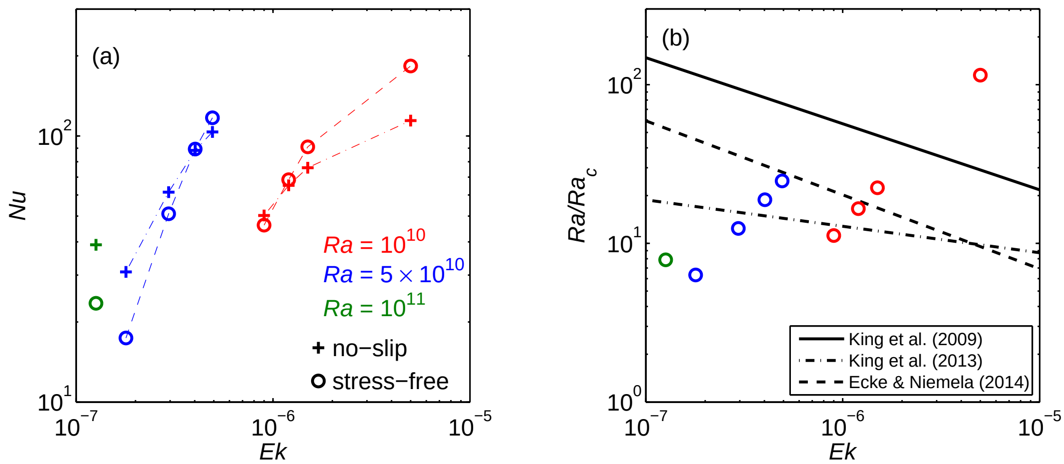

Figure 1. (a) Nusselt number Nu as a function of Ra and Ek for both no-slip and stress-free boundary conditions. (b) Regime diagram in the $(Ek, Ra/Ra_c)$ parameter space after Ref. [2] (Ra_c is the critical Ra at convective onset [1]). Our runs (symbols; same color coding as in (a)) are compared with several predictions of the transition to geostrophic convection: King et al. (2009) [6]; King et al. (2013) [5]; Ecke & Niemela (2014) [2]. The geostrophic regime is found *below* the limiting lines.

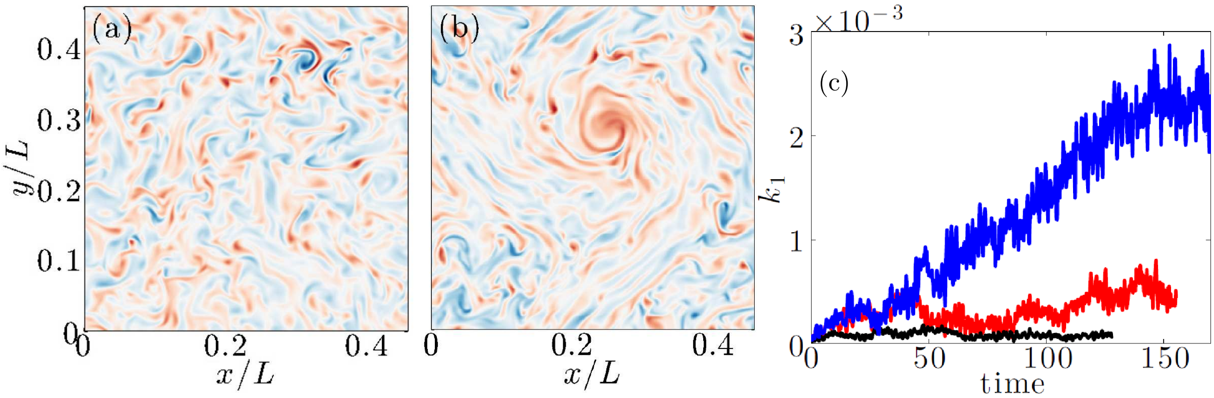

Figure 2. Vorticity snapshots of the mid-plane at $Ra = 10^{10}$ and $Ek = 9 \times 10^{-7}$, where red (blue) indicates positive (negative) vorticity. (a) No-slip plates. (b) Stress-free plates. (c) Growth in time of the amplitude of the Fourier mode with the largest wavelength that fits in the domain: both plates stress-free (blue), top plate stress-free and bottom plate no-slip (red), both plates no-slip (black).

scaling transition is expected upon entry of the geostrophic regime, where Nu becomes more sensitive to rotation (Ek). The current results are in line with this expectation. The exact transition value $Ra_t(Ek, Pr)$ is considered in several recent works [2, 5, 6]. We compare our parameter values to the predicted transitions in figure 1(b). Judging by the slope change of $Nu(Ek)$ alone, the result by Ecke & Niemela [2] appears to match most favorably. However, there is no consensus yet on universal criteria or diagnostics to describe the geostrophic regime. Based on the current results, another criterion could be that the no-slip heat transfer actually supercedes its stress-free counterpart. This remarkable observation must have its origin in the Ekman boundary layers, which are formed near the no-slip plates but not in the stress-free case. The Ekman boundaries have profound effects on the flow structuring.

FLOW STRUCTURING

The striking effect of boundary conditions on the overall flow field is illustrated in figure 2: snapshots of the vertical component of vorticity at mid-plane are plotted for both no-slip (figure 2(a)) and stress-free (figure 2(b)) conditions. For stress-free plates there is a condensation into a large cyclonic vortex with signs of a weaker anticyclonic vortex as well. In the no-slip case no such large structure is found. The growth of this vortex can be observed by defining the horizontal kinetic energy as $k_H = (u_x^2 + u_y^2)/2$ (half the squared sum of the horizontal velocity components u_x and u_y), calculating its 2D Fourier transform, and monitoring in time the amplitude of the principal Fourier mode (the largest mode that fits into the domain). This result is plotted in figure 2(c). The amplitude of the principal mode is growing steadily until $t \approx 140$ for stress-free plates, while for no-slip plates there is only a slight temporal fluctuation. Even a situation with one stress-free and one no-slip plate shows a dramatically reduced amplitude of the principal Fourier mode compared with the pure stress-free case. We infer that the Ekman boundaries are sources of small-scale fluctuations that prevent the condensation into large-scale vortices. Thereby, they also actively counteract the rotational damping of fluctuations which leads to an enhanced overall heat transfer in the no-slip case compared with stress-free.

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