# MEAN FLOW GENERATION DUE TO LONGITUDINAL LIBRATIONS OF SIDE-WALLS OF A ROTATING ANNULUS

Torsten Seelig [1], Abouzar Ghasemi V. [2], Michael Kurgansky [3], Marten Klein [2], Andreas Will [2] & Uwe Harlander [1]

[1] *Department of Aerodynamics and Fluid Mechanics, Brandenburg University of Technology Cottbus-Senftenberg, Cottbus, Germany*

[2] *Department of Environmental Meteorology, Brandenburg University of Technology Cottbus-Senftenberg, Cottbus, Germany*

[3] *A. M. Obukhov Institute of Atmospheric Physics, Russian Academy of Sciences, Moscow, Russia*

Abstract Laboratory experiments with rotating annuli are reported that reveal a prograde jet, which is adjacent either to a (longitudinally) librating inner straight cylinder or to a librating inner truncated cone (frustum), whereas the outer cylindrical wall and bottom and top lids rotate with constant angular velocity. In the frustum case, the jet is located on a straight cylindrical surface which is circumscribed about the frustum and joins the bottom lid. These findings are supported by direct numerical simulations which show good agreement between experimental data and numerical results and, when the centrifugal instability of the Stokes boundary layer near the oscillating sidewall does not set in, highlight the important role of local dynamical processes in the corners, between the inner cylinder and the lids, in producing the prograde jet.

## INTRODUCTION

Many planets or moons perform longitudinal librations and are believed to possess a liquid core (like Mercury and Earth's Moon) or liquid layers in the form of subsurface oceans (like Jupiter's moon Europa or Saturn's moon Enceladus) [1]. Mean flow generation by the libration in longitude has been studied by several authors for different geometries. A geometry which is simpler than that of the spherical shell but contains most of its interesting features relates to rotating cylindrical annuli. Recently, special emphasis has been placed on mean flow excitation in rotating closed annular containers due to longitudinal librations of the cavity's walls. This problem was investigated experimentally, theoretically, and numerically [2]. For a cylindrical container (a full cylinder with an inner wall absent), it was shown that the longitudinal libration produces a retrograde mean flow, due to nonlinearities in the Ekman boundary layers which develop over the librating bottom and top lids of the container. The effect of the oscillating outer cylinder on the mean flow is less clear. Usually, in studies with a cylindrical tank libration, the oscillating Stokes-Stewartson layer at the vertical wall is either treated schematically or neglected [3]. The main purpose of this work is, instead, to focus on the oscillating Stokes-Stewartson boundary layer that forms on a cylindrical annulus' sidewall in libration. To isolate the effect, the lids of the annulus are assumed to rotate with constant angular velocity. That is, we consider a problem, which is in a sense complementary to that of [3].

The work is motivated by (i) possible geophysical and technical applications, (ii) existing challenges of theoretical treatment of the problem, e.g. compared to the case of spherical geometry, and (iii) laboratory experiments with the rotating annuli in librations and numerical simulations of these phenomena, both conducted in Brandenburg Technical University, Cottbus-Senftenberg, Germany.

## EXPERIMENTAL SETUP AND NUMERICAL MODEL

In this article we focus on libration of the inner cylinder (either frustum or straight cylinder), i.e., its angular velocity reads $\Omega_1 = \Omega_0(1 + \epsilon \cos \omega t)$ and the librational forcing is characterized by an additive sinusoidal term of amplitude $\epsilon$ and frequency $\omega$ that could be realized in the laboratory. The remaining walls were kept at a constant rotation rate $\Omega_0$. In the laboratory apparatus shown in Figure 1 the outer cylinder wall is made out of borosilicate glass, the inner frustum is made of aluminum. The alternative straight inner cylinder (schematically dash-dotted in Figure 1) is also made of aluminum. Rigid lids on the top and bottom close the annular gap. The top lid is made of acrylic glass and is connected with the outer cylinder. The bottom lid is made of aluminum, rests on a turntable and serves as support for the outer cylinder. All aluminum parts were anodized to suppress parasitic optical reflections and to make the surface resistant against chemical processes. For more details see [4].

We performed direct numerical simulations (DNS) for the same geometries studied in the laboratory. No-slip conditions are used for all walls. Staggering of dynamical variables ensures that no further regularization of the boundary conditions is needed. [4] performed 2D DNS of inertial waves in an annulus bounded by frustum and a straight cylinder with the same code that was used for the present study and more details on the numerics can be found there.

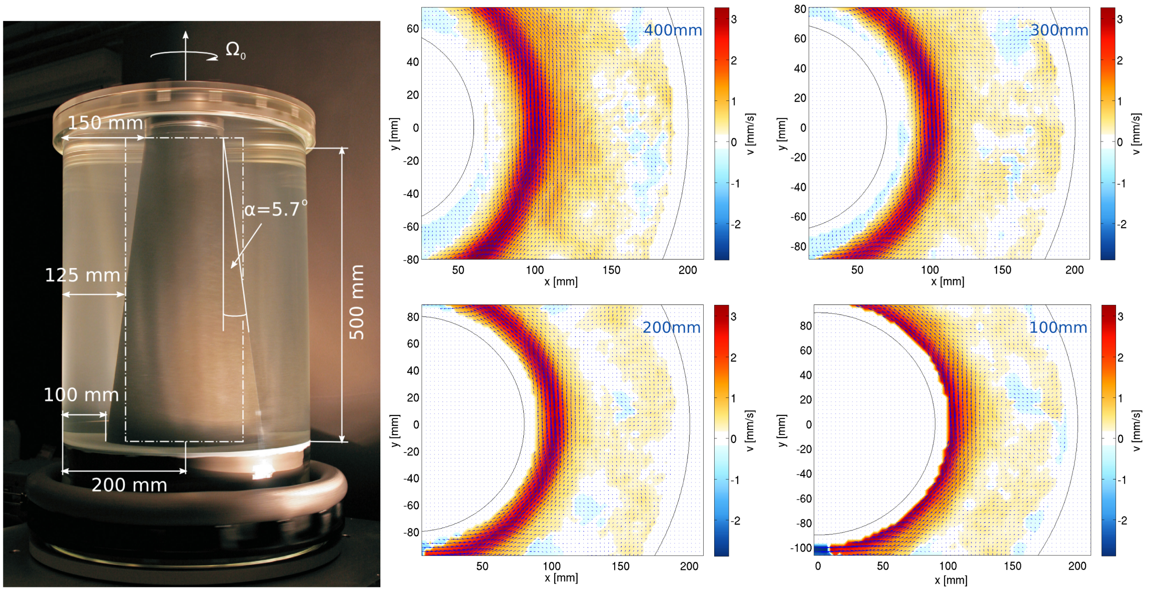


**Figure 1.** Sketch of the experimental setup (left) and PIV measurments (right) for . Shown is the time averaged flow. The prograde jet (red) is clearly visible. The jet forms a cylinder and is thus detached from the frustum in the upper part of the annulus.

## RESULTS

The libration driven jet is shown in Figure 1 (right). Direct numerical simulations show good agreement with experimental results that virtually correspond to a supercritical regime (with respect to the viscous centrifugal instability). The additional virtue of numerical simulations is in highlighting the important role of dynamical processes in the corners, between the inner cylinder and the lids, in producing the prograde jet in a 'globally' laminar regime, i.e., when the centrifugal instability in the Stokes layer near the librating sidewall does not set in [2]. The numerical experiments reveal a significant axial dependence of the flow with the maximum amplitudes near the corners, where the local non-linear processes virtually play the principal formative role, as flow snapshots and animations show. For libration frequencies smaller than 2 we are in a regime where inertial waves can be excited directly by libration. However, in the limit of small libration amplitudes such inertial waves have small amplitude and there is numerical evidence that small amplitude waves do not (notably) interact with the mean flow jet discussed here. Both, the mean flow jet and inertial waves, can be regarded as linearly superposed, as [4] noted for the case of the frustum in libration. However, this approximation breaks down if $\epsilon$ exceeds a certain value. Further research is needed to quantify the wave-mean flow interaction between inertial waves and the emerging jet on the co-axial cylinder that extends into the fluid bulk in case of an inner frustum.

As mentioned above, one of the possible applications of the obtained results may, e.g., be to librations of Galilean moons of Jupiter, believably possessing prominent liquid oceans between their rigid crust and solid core. Moreover, there are certainly other situations in geophysical flows dominated by boundary layer effects in which the presented mean flow generation mechanisms can be of importance.

## References

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