NORMAL MODE DECOMPOSITION IN DIRECT NUMERICAL SIMULATIONS OF ROTATING-STRATIFIED TURBULENCE

Corentin Herbert[1], Raffaele Marino[1,2], Annick Pouquet[1,3] & Duane Rosenberg[4]
[1]*National Center for Atmospheric Research, P.O. Box 3000, Boulder, CO 80307, USA*
[2]*Institute for Chemical-Physical Processes - IPCF/CNR, Rende (CS), 87036, Italy*
[3]*Laboratory for Atmospheric and Space Physics, University of Colorado, Boulder, CO 80309, USA*
[4]*National Center for Computational Sciences, Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, TN 37831, USA*

Abstract In the presence of solid-body rotation and density stratification, turbulent flows may exhibit an inverse as well as a direct cascade. We investigate the role of turbulence and waves in these energy cascades, focusing on the inverse cascade. This is done through a normal mode decomposition of the dynamical fields in a set of direct numerical simulations in terms of inertia-gravity waves and vortical modes. In agreement with theoretical arguments, we find that the vortical modes dominate the inverse cascade of energy.

INTRODUCTION

Turbulent flows subjected at the same time to the effect of solid-body rotation and density stratification are ubiquitous in nature: the atmosphere and the ocean provide examples of such flows. From a theoretical point of view, it is worthy of note that due to the introduction of new timescales, other regimes than the Kolmogorov scenario of homogeneous isotropic turbulence are possible. It has been shown in particular that such flows support at the same time an inverse (upscale) cascade of energy and a direct (downscale) cascade of energy with constant flux [5]. Due to the appearance of linear terms in the Navier-Stokes equations, to take into account the Coriolis and the buoyancy (in the Boussinesq approximation) forces, the system supports the propagation of waves in addition to the usual turbulent motions governed by the nonlinear term. A natural question is then how the interplay between waves and turbulence leads to the phenomenology observed in rotating-stratified flows. We report on an approach using the normal modes of the linearized equations to disentangle the role of the waves and that of turbulent eddies in data generated by direct numerical simulations, focusing on the inverse cascade scenario.

METHODS

We consider incompressible rotating-stratified flows in the Boussinesq approximation:

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla P + \nu\Delta\mathbf{u} - 2\Omega\mathbf{e}_z\times\mathbf{u} - N\theta\mathbf{e}_z + \mathbf{F}_v, \qquad \nabla\cdot\mathbf{u} = 0 \tag{1}$$

$$\partial_t\theta + \mathbf{u}\cdot\nabla\theta = Nu_z + \kappa\Delta\theta, \tag{2}$$

where $f = 2\Omega$ is the Coriolis frequency, N the Brunt-Väisälä frequency and $\mathbf{F}_v$ is a three-dimensional isotropic random forcing acting in the narrowband $k_F \in [22, 23]$. The scalar field is not forced directly. The domain has unit aspect ratio and periodic boundary conditions. The equations are solved numerically using the GHOST (*Geophysical High-Order Suite for Turbulence*) pseudo-spectral code [4], at 512^3 resolution, for various values of the parameters N and f. These simulations feature an inverse cascade of energy [3].
The method of analysis of the numerical data relies on the introduction of an orthonormal basis in Fourier space [1]: $(\mathbf{X}_0(\mathbf{k}), \mathbf{X}_+(\mathbf{k}), \mathbf{X}_-(\mathbf{k}))$ — with $\mathbf{X}_r(\mathbf{k})^\dagger\mathbf{X}_s(\mathbf{k}) = \delta_{rs}$ — which is made of eigenvectors of the linearized dynamics:

$$\dot{\mathbf{X}}_0(\mathbf{k}) = \mathbf{L}(\mathbf{k})\mathbf{X}_0(\mathbf{k}) = 0, \tag{3}$$

$$\dot{\mathbf{X}}_\pm(\mathbf{k}) = \mathbf{L}(\mathbf{k})\mathbf{X}_\pm(\mathbf{k}) = \pm i\sigma(\mathbf{k})\mathbf{X}_\pm(\mathbf{k}). \tag{4}$$

These modes have a natural physical interpretation: the modes $\mathbf{X}_\pm(\mathbf{k})$ are inertia gravity waves, with dispersion relation $\sigma(\mathbf{k}) = \sqrt{f^2k_\parallel^2 + N^2k_\perp^2}/k$, and the mode $\mathbf{X}_0(\mathbf{k})$ corresponds to eddies advected by the nonlinear term. Since the eddy-turnover time is usually much larger than a period of the inertia-gravity waves, these modes are called *slow modes*. Alternatively, they can be characterized as satisfying balance relations: the horizontal pressure gradient compensates the Coriolis force (geostrophic balance) and the vertical pressure gradient compensates the buoyancy force (hydrostatic balance). Besides, they are the modes which contribute to potential vorticity $\Pi = f\partial_z\theta - N\omega_z + \boldsymbol{\omega}\cdot\nabla\theta$ (whereas the wave modes do not). Hence the slow modes can also be referred to as the *balanced modes* or the *vortical modes*.

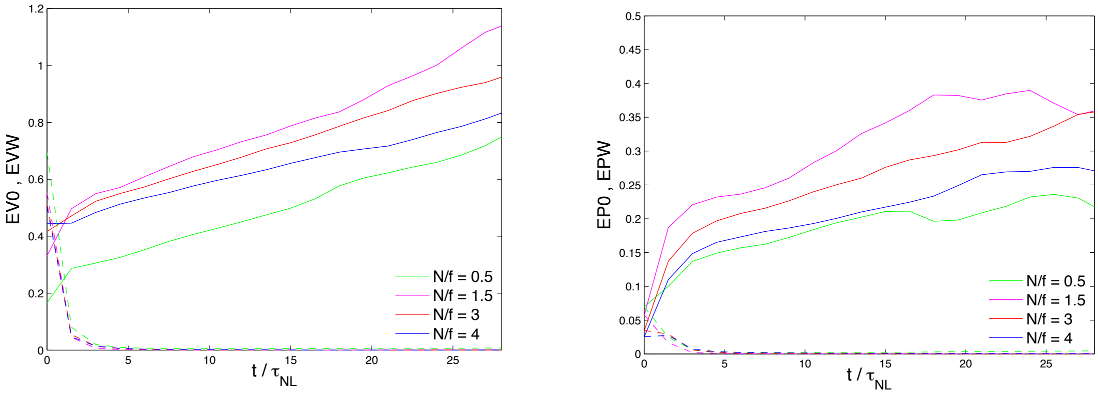

Figure 1. Time evolution of the kinetic energy (left) and the potential energy (right) in the vortical and wave modes, for various N/f ratios. The flow is quickly dominated by the vortical modes.

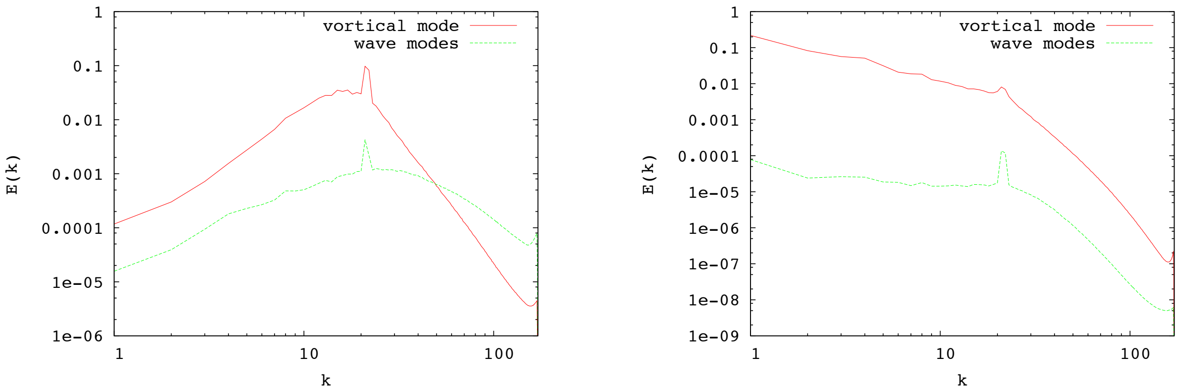

Figure 2. Energy spectra of the vortical and wave modes in a run with $N/f = 1.5$, at an early time (left) and at a late time (right).

RESULTS

We have carried out a decomposition of the numerical data in terms of the normal modes introduced above. Figure 1 shows the evolution in time of the kinetic and potential energy of the wave and vortical modes for different values of the parameters. In all cases, we see that the vortical component rapidly dominates, both in terms of kinetic and potential energy. Besides, the vortical kinetic energy grows with a steady rate, indicating that it is responsible for the observed inverse cascade of energy [3], in agreement with a theoretical prediction based on statistical mechanics [2]. Note that the potential energy of the vortical modes also grows at first, at approximately the same rate as the kinetic energy, but appears to saturate towards the end of some of the runs.

Similarly, we can decompose the energy spectrum as the sum of the energy spectrum of the vortical mode and the energy spectrum of the wave modes. An example is shown in Fig. 2: at early times, the vortical mode dominates the large scales and the wave modes dominate the small scales, but as the inverse cascade builds up, the vortical mode becomes dominant at all scales.

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