PARTICLES AND FIELDS IN SUPERFLUIDS: INSIGHTS FROM THE TWO-DIMENSIONAL GROSS-PITAEVSKII EQUATION

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Abstract We study the dynamics of active particles in two-dimensional superfluids at temperature $T = 0$, for a variety of initial configurations, by carrying out extensive direct-numerical-simulations of the two-dimensional, Galerkin-truncated Gross-Pitaevskii equation. Our study elucidates the interplay of particles and fields, in both simple and turbulent flows. We show that particle collisions can be inelastic, if the repulsive interactions between particles is weak, and elastic otherwise. We show that assemblies of many particles and vortices yield turbulent spatiotemporal evolutions.

INTRODUCTION

We initiate a study of particles in superfluids by developing a new algorithm for particle motion in a superfluid, which is described by the Gross-Pitaevskii (GP) equation [1, 2, 3, 4]. The particles we use are active, i.e., they are not only advected by the flow, but they also act back on it; furthermore, they interact with each other, not only by virtue of superfluid-mediated, effective interactions, but also by repulsive interactions that rise rapidly as the particles approach each other.

RESULTS

Our study elucidates the interplay of particles and fields in superfluids, in both simple and turbulent flows, and yields several new results, which we summarize below: At the one-particle level we explore, for light, neutral, and heavy particles, the nature of their dynamics in the superfluid, when a constant external force acts on them; in particular, we explore how the particle motion can become chaotic. We show that the interaction of the particle(s) with vortices leads to dynamics that depends sensitively on the particle characteristics. We extend these studies to the case of two particles, where we show the existence of an effective, superfluid-mediated, attractive interaction between the particles. Moreover, we introduce a short-range repulsive interaction between particles and show how collisions of heavy, neutral, and light particles are different. Here, we find that, at low values of the range of the repulsive force, the collisions are completely inelastic (Fig. 1(a)), with coefficient of restitution equal to zero; as we increase the range of the repulsive force the coefficient of restitution becomes finite at a critical point, and finally attains values close to unity, i.e., the collisions become elastic (Figs. 1(b) and 1(c)). Furthermore, we find that many-particle collision dynamics also depends on the range of the repulsive force. At large values of the range of the repulsive force, we obtain chaotic, many-particle collision dynamics. Our studies of assemblies of particles and vortices demonstrates that their dynamics show rich, turbulent spatiotemporal evolution (Figs. 2(a)-(f)).

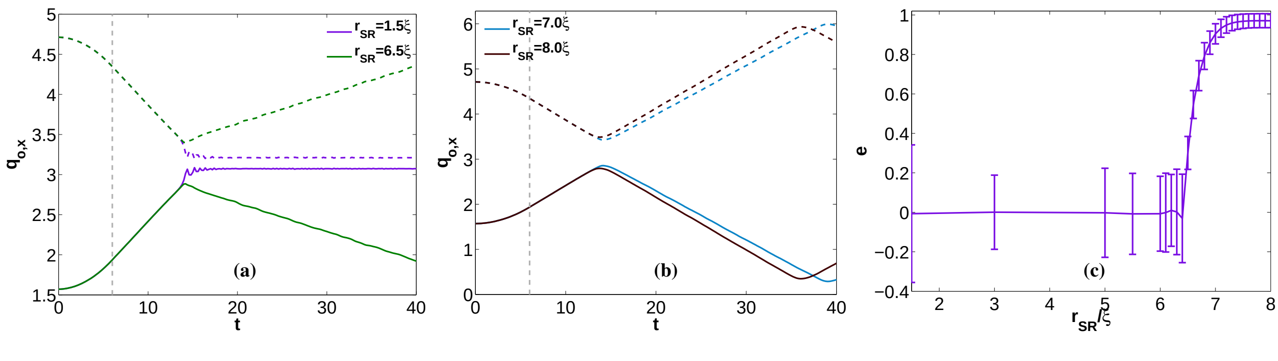

Figure 1. Head-on collisions between two neutral particles: (a) particle trajectories shown via plots of the x-component of the particle position vector $q_{o,x}$ versus time and $r_{\rm SR} = 1.5\xi$ (purple curve) and $r_{\rm SR} = 6.5\xi$ (green curve); (b) particle trajectories shown via plots of the x-component of the particle position vector $q_{o,x}$ versus time and $r_{\rm SR} = 7.0\xi$ (sky-blue curve) and $r_{\rm SR} = 8.0\xi$ (brown curve). (c) Plot of the coefficient of restitution e versus $r_{\rm SR}/\xi$, for a head-on collision between two neutral particles in our 2D, Gross-Piteavskii superfluid. $r_{\rm SR}$ is the range of the short-range, repulsive force between two particles and ξ is the healing length, which can be associated with the vortex-core size.

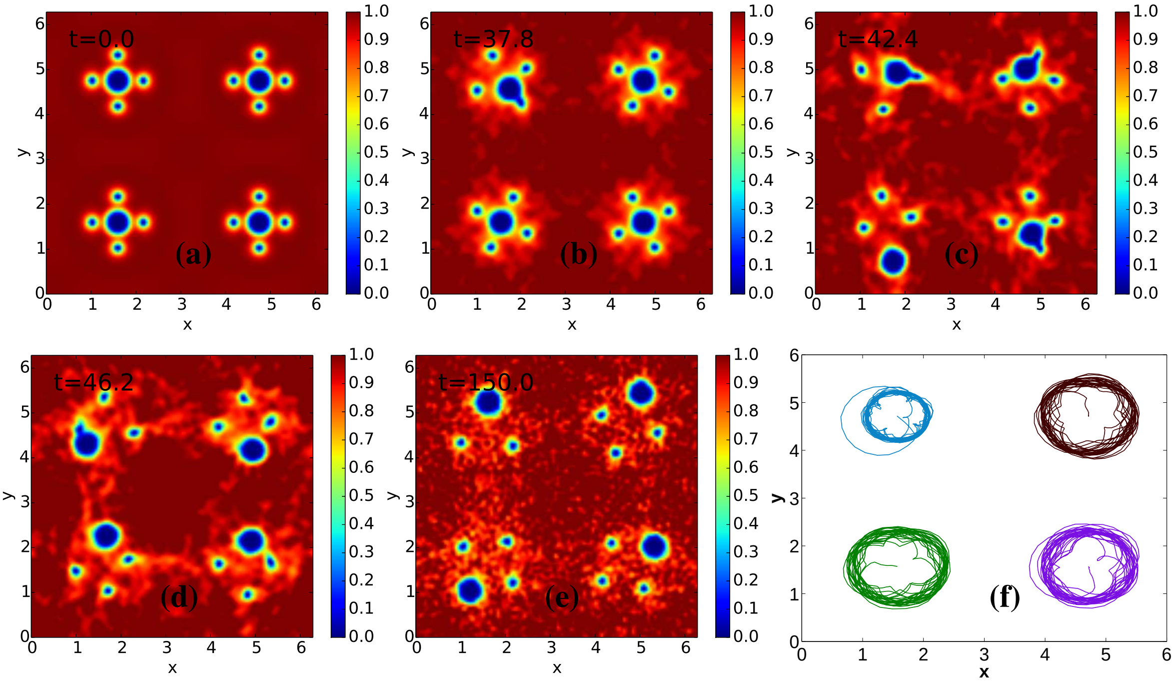

Figure 2. (a)-(e) Spatiotemporal evolution of the field $|\psi(\mathbf{x},t)|^2$ shown via pseudocolor plots, for four neutral particles (large blue patches), initially placed at the centers of the counter-rotating vortex clusters, at the coordinates $(3\pi/2, \pi/2)$, $(\pi/2, \pi/2)$, $(\pi/2, 3\pi/2)$, and $(3\pi/2, 3\pi/2)$, respectively, at $t = 0$. $\psi(\mathbf{x}, t)$ is the complex field describing the superfluid in the GP theory. (f) Trajectories of the four neutral particles in the presence of counter-rotating vortex clusters, shown by purple, green, blue, and brown curves.

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