LOGARITHMIC VARIANCE PROFILES AND THE CORRESPONDING $f^{-1}$ SPECTRA OF TEMPERATURE FLUCTUATIONS IN TURBULENT RAYLEIGH-BÉNARD CONVECTION

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Abstract We report experimental results for the temperature variance $\sigma^2(z)$ and the corresponding frequency spectra $P(f)$ in turbulent Rayleigh-Bénard convection (RBC) in a cylindrical sample of aspect ratio $\Gamma = D/L = 1.00$ ($D = 1.12$ m is the diameter and $L = 1.12$ m the height). The measurements were conducted in the Rayleigh-number range $10^{11} \leq Ra \leq 1.35 \times 10^{14}$ and Pr $\simeq 0.8$. For $Ra = 1.35 \times 10^{14}$, $\sigma^2(z)$ could be described well by a logarithmic dependence on the vertical position $z$ in a range of $z^\ast \leq z \leq z^\ast_2$ with $z^\ast_1 \simeq 70\lambda_0$ and $z^\ast_2 = 0.1L$. Here $\lambda_0 \equiv L/(2Nu)$ is the thickness of a thin thermal sublayer adjacent to the horizontal plate where the heat flux (denoted by the Nusselt number $Nu$) is carried mostly by thermal diffusion. In the log layer, we found that the temperature spectra had a significant frequency range over which $P(f) \sim f^{-\alpha}$ with $\alpha$ close to 1. As $Ra$ decreased, $\lambda_0$ increased so that the log layer became thinner. At $Ra = 2.05 \times 10^{11}$, $z^\ast_2 \lesssim z^\ast_1$ and therefore there was no range for a log layer. Correspondingly, the temperature spectrum near the horizontal plate did not have the $f^{-1}$ scaling form either.

Details about the RBC sample and experimental procedures were reported in Refs. [1, 2]. In the present work, we installed 68 new thermistors to measure temperature fluctuations. These thermistors were positioned in 6 columns at various radial locations $r$ from 1.0 cm to 15.0 cm away from the side wall within the sample. The thermistor diameters were 0.36 mm. The vertical positions of the thermistors were distributed over a range of $0.013 \leq z/L \leq 0.990$, symmetrically about the mid-height of the sample. They were known with a precision of 1 mm. The sample was carefully leveled relative to gravity to within $10^{-4}$ rad. For temperature spectral measurements we used an ac bridge and a lock-in amplifier for each thermistor. Each amplifier was operated at a working frequency in the range $f_0 \sim 1 \pm 0.4$ kHz to measure temperatures at a rate of 40 Hz.

Figure 1 shows the results for the temperature variance profiles $\sigma^2(z)$ at the radial position $\xi = 0.064$ for different $Ra$. The vertical position $z$ is scaled by the length $\lambda_0 \equiv L/(2Nu)$. Here $\lambda_0$ is the thickness of a thin thermal sublayer adjacent to the horizontal plate where the heat flux is carried mostly by thermal diffusion. This thermal sublayer in RBC plays a role similar to the viscous sublayer in wall-bounded shear flow. At the highest $Ra = 1.35 \times 10^{14}$, the data follow closely a logarithmic dependence on the vertical position $z$ in a range of $z^\ast_1 \simeq z \leq z^\ast_2$ with $z^\ast_1 \simeq 70\lambda_0$ and $z^\ast_2 = 0.1L$. When $Ra$ decreases, $\lambda_0$ increases and the log-layer upper limit $z^\ast_2/\lambda_0$ decreases. As a result, the log-layer range becomes smaller. At $Ra = 2.05 \times 10^{11}$, $z^\ast_2/\lambda_0 \lesssim z^\ast_1/\lambda_0$ and therefore there is no range for the log layer of $\sigma^2(z)$.

In Fig. 2 we show the compensated temperature frequency spectra $(fT_0) \times P(fT_0)$ as a function of the normalized frequency $fT_0$ measured at $z/L = 0.019$ and $\xi = 0.064$ for different $Ra$. Here $T_0$ is a characteristic time scale determined from the temperature auto-correlation function [3]. The two spectra, although measured at the same distance from the bottom plate, correspond to different $z/\lambda_0$ because of different $Ra$. For $Ra = 1.35 \times 10^{14}$ the measuring positions is inside the log layer with $z/\lambda_0 \sim 101$. In the low-frequency range $0.02 \leq fT_0 \leq 0.2$ the compensated spectrum has the scaling $P(fT_0) \sim (fT_0)^{-\alpha}$ with $\alpha \simeq 1$, as indicated by a plateau of $(fT_0) \times P(fT_0)$. This spectral scaling form and the corresponding logarithmic profile are consistent with previous measurements for $z/L \lesssim 0.1$ in a $\Gamma = 0.50$ sample with $Ra$ above $1.63 \times 10^{13}$ [3]. For $Ra = 2.05 \times 10^{11}$ the measuring position corresponds to $z/\lambda_0 \sim 12.7$. Because there is no log layer as shown in Fig. 1 (c), the corresponding spectrum does not have the $f^{-1}$ scaling. These temperature variance profiles and the corresponding frequency spectra in turbulent RBC share many similarities with predictions for the variance profiles and the wave-number spectra of velocity fluctuations in the log layer of turbulent pipe flow $[4, 5]$

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References

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Figure 1. (a) Measured temperature variance $\sigma^2(z)$ as a function of the normalized vertical position $z/\lambda_\theta$ on a logarithmic horizontal scale for the three Rayleigh numbers (a) $Ra = 1.35 \times 10^{14}$, (b) $4.38 \times 10^{12}$, and (c) $2.05 \times 10^{11}$. The vertical solid lines are at $z/\lambda_\theta = 70$. Three vertical dashed lines represent $z/L = 0.1$. The red solid line in (a) is a fit to the data for $z/L < 0.1$ using the logarithmic function $\sigma^2(z,r) = M(r) \cdot \ln(z/L) + N(r)$. All measurements were for the normalized radial location $\xi \equiv (R - r)/R = 0.064$.

Figure 2. Normalized temperature spectra $(f\tau_0) \times P(f\tau_0)$ as a function of $f\tau_0$ at $z/L = 0.019$ for $Ra = 1.35 \times 10^{14}$ (black solid line) and $2.05 \times 10^{11}$ (red dashed line). All measurements were for the normalized radial location $\xi \equiv (R - r)/R = 0.064$. 