NONLINEAR DYNAMICS OF LARGE-SCALE COHERENT STRUCTURES IN FREE SHEAR LAYERS

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Abstract
It is well known that fully developed turbulent free shear layers exhibit a high degree of order, characterized by large-scale coherent structures, i.e. spanwise vortex rollers. Extensive experimental investigations show that such organised motions bear remarkable resemblance to inviscid instability waves, and their main characteristics, including the length scales, propagation speeds and transverse structure, are reasonably well predicted by inviscid linear stability analysis of the mean flow. In this paper, we present a mathematical theory to describe the nonlinear dynamics of coherent structures. The theory is adapted from the nonlinear non-equilibrium critical-layer approach for laminar-flow instabilities by accounting for (a) the enhanced non-parallelism associated with fast spreading of the mean flow, and (b) the influence of small-scale turbulence on coherent structures. The combination of these factors with nonlinearity leads to an interesting evolution system, consisting of the coupled amplitude and vorticity equations, in which non-parallelism contributes the so-called translational critical-layer effect. Numerical solutions of the evolution system captures vortex roll-up, which is the hallmark of turbulent mixing layer, and the predicted amplitude development closely mimics what was measured in experiments.

Key words: turbulence, coherent structures, instability, nonlinearity

INTRODUCTION AND SKETCH OF A NONLINEAR THEORY

It has been widely recognized since 1970s that orderly and quasi-deterministic fluctuations, characterized by coherent structures (CS), are present in turbulent shear flows. Brown & Roshko (1974) provided the first visualizations of CS in a high-Reynolds-number plane turbulent mixing layer. The striking pictures showed clearly that CS were predominantly two dimensional and consisted of an array of spanwise concentrated vortices. These vortices propagate at a constant speed, which is approximately the average of the free stream velocities. The origin and development of CS have been extensively investigated for several decades. It has long been suggested that CS are Kelvin-Helmholtz vortices arising from small perturbations as a result of instability of the mean flow. Experiments using controlled excitations have provided much information about the kinematics and dynamics of CS. Gaster, Kit & Wygnanski (1985) carried out detailed measurements for a relatively low level excitation. The characteristics wavelength, propagation speed and transverse distribution of CS were found to correspond to those of the linear instability mode of the mean flow, but the growth rate exhibited considerable discrepancy. Fiedler & Mensing (1985) investigated CS forced by a periodic disturbance of various frequency and amplitude. For weak excitations, the evolution of CS with different frequencies follows a universal rule. For strong excitations, the decay is oscillatory. Weisbrot & Wygnanski (1988) found that the transverse distribution of the velocities and Reynolds stress were well predicted by the linear stability analysis of the mean flow even at high level excitation.

In this paper, we propose a theory for the nonlinear development of CS. The instantaneous field is composed of a mean flow $(\bar{\mathbf{U}}, \bar{P})$, coherent motion $(\tilde{\mathbf{u}}, \tilde{p})$ and small-scale turbulence $(\mathbf{u}', p')$, and thus has the triple decomposition

$$(\mathbf{u}, p) = (\bar{\mathbf{U}}, \bar{P}) + (\tilde{\mathbf{u}}, \tilde{p}) + (\mathbf{u}', p'). \tag{1}$$

Unlike the conventional treatment (Hussain & Reynolds 1972), here we take $(\bar{\mathbf{U}}, \bar{P})$ to be the 'partial mean flow' driven only by the Reynolds stresses of $(\mathbf{u}', p')$, while the mean flow generated by $(\tilde{\mathbf{u}}, \tilde{p})$ is to be treated as part of CS.

For the time-averaged Reynolds stresses, the model is, in the non-dimensional form, written as

$$\bar{\tau}_{ij} = \overline{u_i' u_j'} = -\frac{1}{R_T}(\partial \bar{U}_i/\partial x_j + \partial \bar{U}_j/\partial x_i),$$

where the mean turbulent Reynolds number $R_T = \rho U\delta/\mu_t$ with $\bar{\mu}_t$ being a mean eddy viscosity. The phase-averaged Reynolds stresses are time dependent, and they are related to the time-dependent strain rate of CS by a gradient type of model that includes the effect of time relaxation (Wu & Zhou 1989), that is,

$$\tilde{\tau}_{ij} = < u_i' u_j' > - \overline{u_i' u_j'} = -\frac{1}{\tilde{R}_T}(\partial \tilde{u}_i/\partial x_j + \partial \tilde{u}_j/\partial x_i)(\mathbf{x}, t - \hat{\tau}), \tag{2}$$

where $\tilde{R}_T = \rho U\delta/\tilde{\mu}_t$ with $\tilde{\mu}_t$ being an eddy viscosity accounting for the impact of small-scale turbulence on CS.

Coherent structures are to be represented by a dominant instability mode with a frequency. Following the initial exponential growth, it becomes neutral at some streamwise location, in the vicinity of which the disturbance enters a nonlinear stage due to enhanced nonlinearity associated with emergence of a critical layer as in the laminar flows.

In the main part of the shear layer, the disturbance may be written as (Wu & Tian 2012)

$$\epsilon \bar{A}(\bar{x}, \tau)\, \mathrm{e}^{\mathrm{i}\,\bar{\zeta}} \quad \text{where} \quad \bar{\zeta} = \alpha(x - ct), \quad \bar{x} = \epsilon^{\frac{1}{2}} c^{-1} x, \quad \tau = \epsilon^{\frac{1}{2}} t, \tag{3}$$

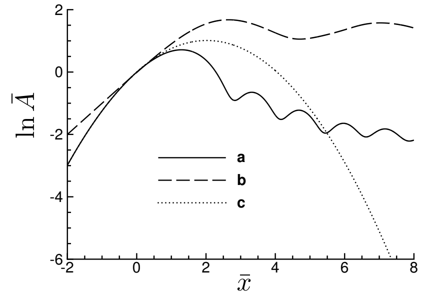

Figure 1. Nonlinear development of the amplitude $\bar{A}$ (solid line (a)). Dashed line (b): parallel-flow approximation; dotted line (c): linear evolution.

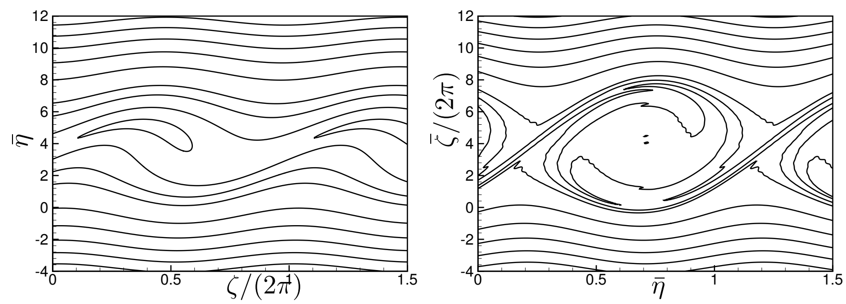

Figure 2. Vortex roll-up as shown by contours of critical layer vorticity at $\bar{x} = 0$ and $\bar{x} = 2$.

where α and c are the wavenumber and phase speed of the locally neutral mode respectively, and $\bar{A}$ is the amplitude function of $\bar{x}$ and τ. Within the critical layer, the effects of non-equilibrium, viscosity and nonlinearity all come into play at leading order under the distinguished scalings that $\tilde{R}_T = O(R) = O(\epsilon^{3/2})$. Crucially, we take $R_T = O(R^{2/3})$ so that the non-parallel-flow effect appears also at leading order. We write

$$R^{-1} = \bar{\lambda}_1 \epsilon^{3/2}, \quad \tilde{R}_T^{-1} = \bar{\lambda}_2 \epsilon^{3/2}, \quad \sigma R_T = R^{2/3}. \tag{4}$$

The nonlinear evolution of CS is governed by the coupled system for the amplitude $\bar{A}$ and the critical-layer vorticity Ω

$$\left.\begin{aligned} &\Big[\frac{\partial}{\partial\tau} + \frac{\partial}{\partial\bar{x}} + \bar{\eta}\frac{\partial}{\partial\bar{\zeta}} - (\mathrm{i}\,\bar{A}\,\mathrm{e}^{\mathrm{i}\,\bar{\zeta}} + c.c. - \chi)\frac{\partial}{\partial\bar{\eta}} - \bar{\lambda}_1\frac{\partial^2}{\partial\bar{\eta}^2}\Big]\Omega - \bar{\lambda}_2\frac{\partial^2}{\partial\bar{\eta}^2}\Omega(t-\hat{\tau}) = \Big[-(\frac{\partial}{\partial\tau} + \frac{\partial}{\partial\bar{x}}) + \mathrm{i}\,\chi_2\bar{x}\Big](\bar{A}\,\mathrm{e}^{\mathrm{i}\,\bar{\zeta}} + c.c.),\\ &\qquad\qquad \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_0^{2\pi}\Omega\,\mathrm{e}^{-\mathrm{i}\,\bar{\zeta}}\,\mathrm{d}\,\bar{\zeta}\,\mathrm{d}\,\bar{\eta} = \Lambda_1\frac{\partial\bar{A}}{\partial\tau} + \Lambda_2\frac{\partial\bar{A}}{\partial\bar{x}} + \Lambda_0\bar{x}\bar{A}. \end{aligned}\right\} \tag{5}$$

MAIN RESULTS

The nonlinear evolution is solved numerically for typical values of the parameters. The amplitude evolution is shown in figure 1. Under the influence of both nonlinearity and non-parallelism, CS saturate and delay in an oscillatory manner as observed in experiments. In contrast, when nonlinearity is ignored, CS decay rapidly and monotonically (dashed line) while when non-parallelism is neglected, the amplitude increases slightly. Figure 2 shows that in the initial linear stage, the vorticity field exhibits a simple cat's eye pattern. As nonlinearity comes into play, the vorticity in the critical layer rolls up to form concentrated vortices.

It may be concluded that the present theory is able to predict two key dynamic characteristics of CS: oscillatory decay of their amplitudes, and formation of vortex rollers. Further results will be presented at the conference.

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