MULTIFRACTAL DROPLET DYNAMICS IN TWO-DIMENSIONAL, BINARY-FLUID TURBULENCE

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Abstract We present the most extensive direct numerical simulations, attempted so far, of statistically steady, homogeneous, isotropic turbulence in two-dimensional, binary-fluid mixtures with air-drag-induced friction. We model this mixture by using the Cahn-Hilliard-Navier-Stokes equations and choose parameters, e.g., the surface tension, such that we have a droplet of the minority phase moving inside a turbulent background of the majority phase. Our study reveals that a single droplet, whose mean radius lies in the inertial range of scales, (a) enhances the the forward-cascade part of the energy spectrum of two-dimensional turbulence and (b) stretches the tails of the PDF of the Okubo-Weiss parameter Λ. We show that the dynamics of the droplet is affected significantly by the turbulence in the fluid. In particular, the PDFs of the components of the acceleration shows wide, non-Guassian tails. We characterize the time dependence of the deformation of the droplet and show that it exhibits multifractality.

INTRODUCTION

Binary-fluid mixtures have played an important role in the understanding of the statistical mechanics of critical phenomena at the consolute point above which the two fluids mix [see, e.g., Ref. [1, 2]], of the nucleation of droplets below the coexistence curve [see, e.g., Ref. [3]], of spinodal decomposition [4], and of the late stages of phase separation [see, e.g., Ref. [5]]. It has also been recognized that turbulence in such binary-fluid mixtures can modify many aspects of phase separation and the motion of droplets of one phase in the other. Here we investigate how the motion of an active droplet, of the minority phase, modifies the statistical properties of turbulence in a binary-fluid mixture and also how the dynamics of such a droplet is affected by this turbulence.

RESULTS

Our study, which is based on an extensive direct-numerical simulation (DNS), of the two-dimensional Cahn-Hilliard-Navier-Stokes equations [see, e.g., Ref. [6]], in a parameter regime in which a single droplet moves inside a turbulent fluid, yield very interesting results that we summarize below. We find that the fluid energy spectrum $E(k)$ is modified in two important ways by the droplet : (1) $E(k)$ shows oscillations whose period is related inversely to the mean radius of the droplet; (2) the large-k tail of $E(k)$ is enhanced by the droplet (Fig.1(a)). (In the absence of this droplet, our forcing scheme yields a fluid-energy spectrum that is dominated by a forward cascade of the enstrophy.)

To characterize the effect of the droplet on the topology of the flow-field, we plot the probability distribution function (PDF) of the Okubo-Weiss parameter $\Lambda \equiv (\omega^2 - \sigma^2)/8$, where ω is the vorticity and $\sigma^2 = \sum_{ij} \sigma_{ij}\sigma_{ij}$ and $\sigma_{ij} = \partial_i u_j + \partial_j u_i$, where u_i and u_j are the i and j components of the fluid velocity, respectively; $\Lambda > 0$ ($\Lambda < 0$) in vortical (extensional) regions of the flow. We show in Fig.1(b) that the tails of the PDF of Λ fall less rapidly with $|\Lambda|$ than they do in the absence of the droplet.

To investigate the effect of fluid turbulence on the motion of the droplet, we obtain the PDF of the components of acceleration of the centre of mass of the droplet. This PDF shows wide tails (Fig.1(c)) and is reminiscent of the acceleration-component PDFs of Lagrangian tracers in turbulent flows [7]. We also obtain the deformation $\delta = \frac{S(t)}{S_0(t)} - 1$ of the droplet, where, $S(t)$ is the perimeter of the droplet at time t and $S_0(t)$ is the perimeter of undeformed droplet, of equal area, at time $t = 0$. Figure 2(a) shows that the PDF of δ has wide tails when the surface tension is low. We give the temporal evolution of δ in Fig. 2(b). This shows intermittent spikes whose multifractality we characterize by obtaining the singularity spectrum, which we show in Fig. 2(c).

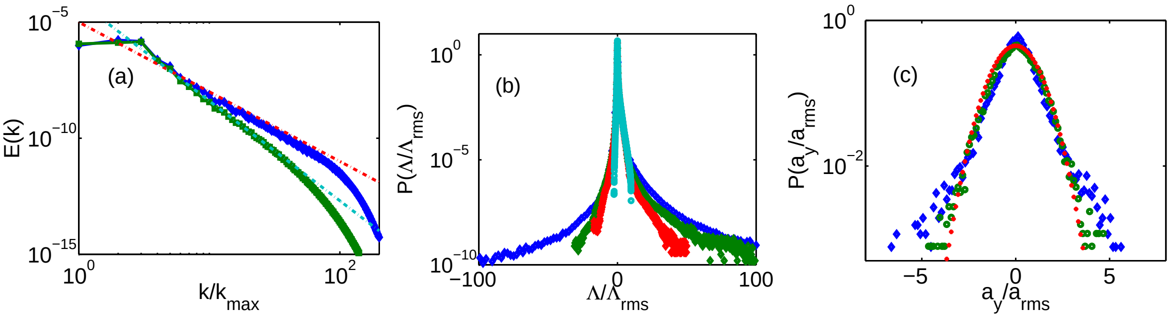

Figure 1. (Colour online) (a) Log-log plots (base 10) versus the scaled wavenumber $\frac{k}{k_{max}}$ of the energy spectra $E(k)$ for $\nu = 10^{-5}$, with a droplet with $a_0 = 120, \beta = 0.025$ (blue diamonds), single phase fluid (green circles), power-law scaling $k^{-3.0}$ (red dashed line) and $k^{-4.3}$ (light blue dashed line) ; (b) PDFs of the Okubo-Weiss parameter Λ for $a_0 = 120, \beta = 0.016$ (blue diamonds), $a_0 = 120, \beta = 0.037$ (green diamonds) $a_0 = 120, \beta = 0.062$ (red diamonds) and single phase fluid (light blue circles); (c) PDFs of a_y of the center of mass of the $a_0 = 80, \beta = 0.037$ droplet (blue circles), the $a_0 = 120, \beta = 0.037$ droplet (green circles) and a Gaussian curve (red circles). Here a_0 is the radius of the initial, undeformed droplet, β is the surface tension parameter, and ν is the kinematic viscosity.

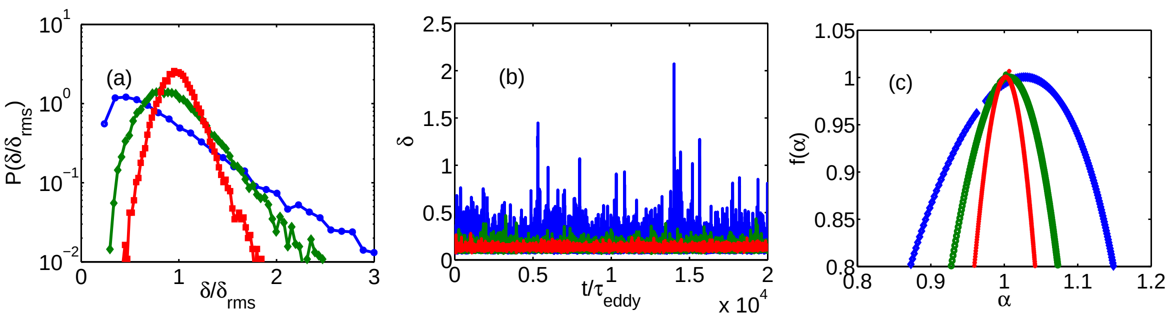

Figure 2. (Colour online) (a) PDFs of δ of the $a_0 = 120, \beta = 0.016$ droplet (blue line), $a_0 = 120, \beta = 0.037$ droplet (green line), and $a_0 = 120, \beta = 0.062$ droplet (red line); (b) plots versus time t/T_{eddy} of the droplet deformation δ of the $a_0 = 120, \beta = 0.016$ droplet (blue line), $a_0 = 120, \beta = 0.037$ droplet (green line), and $a_0 = 120, \beta = 0.062$ droplet (red line); (f) singularity spectra $f(\alpha)$ versus α of the $a_0 = 120, \beta = 0.016$ droplet (blue diamonds), $a_0 = 120, \beta = 0.037$ droplet (green circles), and $a_0 = 120, \beta = 0.062$ droplet (red circles).

Acknowledgements:

We thank CSIR, UGC, DST (India) and SERC (IISc) for computational resources. NP thanks TIFR, Hyderabad for hospitality and PP thanks IISc, Bangalore for hospitality.

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