HYDRODYNAMICAL TURBULENCE BY FRACTAL FOURIER DECIMATION

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Abstract We present a systematic numerical investigation of high-resolution 3D isotropic and homogeneous turbulence resolved on a decimated set of Fourier modes. Fractal decimation acts to decrease the effective dimensionality of the flow by allowing triadic interactions only in a set of Fourier modes $N(k)$ proportional to k^{D_F} for large k. While keeping the symmetries of the original $3D$ Navier-Stokes equations unchanged, a dramatic change in small-scale statistics is detected at decreasing the fractal dimension D_F. Already at fractal dimension $D_F = 2.8$, a global self-similar behaviour is observed in the inertial range of scales, the consequence of such transition are the restoration of the scaling symmetry and vorticity distribution that becomes close to Gaussian.
We relate the results to the different roles of local vs non-local interactions in the energy transfer range.

FROM SCALING-INVARIANCE SYMMETRY BREAKING TO RESTORATION

The understanding of turbulence has considerably advanced in recent years. From the observations collected by means of experiments and direct numerical simulations, we have progressed to the first analytical description of anomalous scaling laws in the problem of turbulent transport [1]. A similar understanding still lacks for the $3D$ Navier-Stokes equations, despite the tremendous collection of attempts [2, 3]. The building bricks of any theory are the concepts of *energy cascade, symmetry invariance and small-scale universality*.
Recently, it has been shown that hydrodynamical turbulence can exist in non-integer space dimension D_F, with the non-linear term conserving energy and enstrophy and in the original problem [4]. Starting from $D_F = 2$, the fractally decimated Navier-Stokes equations exhibit a Gibbs equilibrium state with energy spectrum $E(k) \propto k^{-5/3}$ at the critical dimension $D_F = 4/3$ [5]: morevover the negative energy flux associated to the inverse energy cascade linearly vanishes near the critical dimension $D_F = 4/3$. This supports the interpretation of the inverse energy cascade in two-dimensional turbulence as an equilibrium regime.
What happens if we consider a turbulent flow in the regime of direct energy cascade, starting from $D_F = 3$?

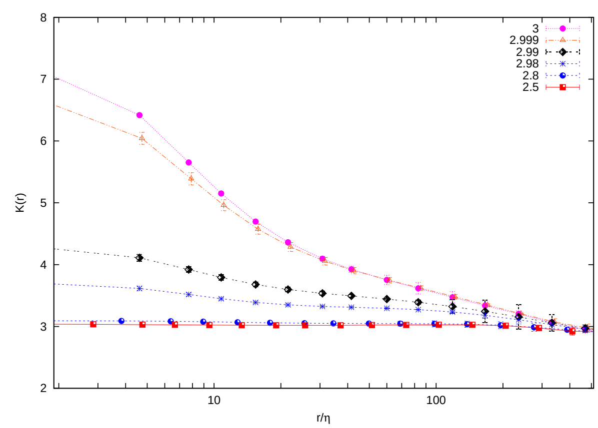

Figure 1. Log-lin plot of the flatness $K(r)$ measured from the longitudinal velocity increments vs the scale separation, normalized by the Kolmogorov scale. Data refer to the set of direct numerical simulation with 1024^3 grid points. Label refer to the simulations at changing the fractal dimension D_F.

To answer this question we performed a series of Direct Numerical Simulations of fractally decimated Navier-Stokes equations with 1024^3 and 2048^3 grid points. The flow is stationary, statistically isotropic and homogeneous. Two kinds of forcings were used [6], the first acting on a narrow band about the smallest wavenumber, the second acting on a larger band of wavenumbers. Fractal decimation is realised by applying a Galerkin truncation in Fourier space and by a *random* decimation of all sub-sets of wave-numbers. As a result, on average $N(k) \propto k^{D_F}$ modes are left in a sphere of radius k.
We explored the range with $2.5 \leq D_F \leq 3$ (see Table 1): this implies that, e.g. at resolution 2048^3 and fractal dimension $D_F = 2.5$, roughly only the 2% of the Fourier modes will be active with respect to the standard three-dimensional case. However, in contrast with the inverse cascade regime, where the energy injection is more and more balanced by the direct dissipation near injection, here the positive energy flux does not vanish when increasing the Fourier decimation.

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D_F	N^3	$\eta/\Delta x$	ν	$\#T_E$	$Forcing$
3	1024^3	0.75	6.e-4	10	F1
3	1024^3	0.95	8.e-4	6	F1
2.999	1024^3	0.8	$6.e-4$	10	F2
2.999	1024^3	0.75	$6.e-4$	15	F1
2.99	1024^3	0.98	$8.e-4$	10	F2
2.99	1024^3	0.8	$6.e-4$	10	F2
2.99^*	1024^3	0.95	$6.e-4$	15	F1
2.99	2048^3	0.7	$2.e-4$	7	F1
2.98	1024^3	1	$8.e-4$	8	F2
2.98	1024^3	0.75	$6.e-4$	10	F2
2.98	1024^3	0.75	$6.e-4$	15	F1
2.98	2048^3	0.7	$2.e-4$	8	F1
2.8	1024^3	0.9	$6.e-4$	10	F1
2.5^*	1024^3	0.65	$6.e-4$	8	F1
2.5^*	1024^3	0.2	$1.5e-4$	8	F1

Table 1. Parameters of the numerical simulations: D_F fractal dimension; grid resolution N^3; Kolmogorov length scale η in simulation units (SU) divided by grid spacing $\Delta x = 2\pi/N$; kinematic viscosity ν (SU); extension of the statistical database in terms of the estimated number ot large-scale eddy turnover times $\#T_E$; F1 forcing at low shells: wavenumbers forced in the range $(0.5 < |\mathbf{k}|^2 < 3)$; F2 forcing acts in the range: $(0.5 < |\mathbf{k}|^2 < 6.25)$. Runs with the symbol $*$ are those repeated with two different realizations of the random realization of the fractal Fourier decimation.

Figure 1 shows the steady-state measure of the velocity field flatness, measured from longitudinal Eulerian moments, as a function of scale r. The consequence of Fourier decimation is dramatic: at $D_F = 2.99$, the small scale flatness has already become 60% smaller, at $D_F = 2.8$ the flatness is equal to its Gaussian value at all scales. Transition to a Gaussian statistics has already been found in turbulent flows, but in the presence of a random stirring with power-law spectrum [7], in agreement with renormalization group predictions [8].
Finally, we find that Fractal decimation also modifies the energy spectrum slope, and the statistics of velocity gradients. In the extreme situations when the system is Gaussian at all scales, it appears that the energy transfer becomes "diffusive-like" with local and non-local triadic interactions playing similar roles.

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