ASPECT-RATIO DEPENDENCE OF THE TRANSITION TO THE ULTIMATE STATE OF TURBULENT RAYLEIGH-BÉNARD CONVECTION

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Abstract We report on measurements of the ultimate-state transition in turbulent Rayleigh-Bénard convection obtained in a large facility known as the "Uboot of Göttingen" and using pressurized sulfur hexafluoride as the convecting fluid. We found that the transition occurs over a range of Ra which becomes more narrow as Γ increases, ranging from Ra_1^* which is at most weakly dependent on Γ and close to 10^{13} to Ra_2^* which varies from about 2×10^{15} for $\Gamma = 0.33$ to about 7×10^{13} for $\Gamma = 1.00$.

Turbulent convection of a fluid heated from below, an important process in numerous geo- and astro-physical systems, is often studied in a fluid contained between parallel horizontal plates (Rayleigh-Bénard convection or RBC). In that case most of the applied temperature difference is sustained by thin boundary layers (BLs) adjacent to the top and bottom plate, and the bulk of the fluid has a temperature that is nearly constant in the time average although it is fluctuating vigorously [1]. When the Rayleigh number Ra is not too large, the BLs are laminar albeit time dependent. It was predicted [7] that the laminar BLs would become turbulent when Ra reaches a critical value Ra^*, with $Ra^* \simeq 10^{14}$ [3] in the case where the Prandtl number Pr is close to one.

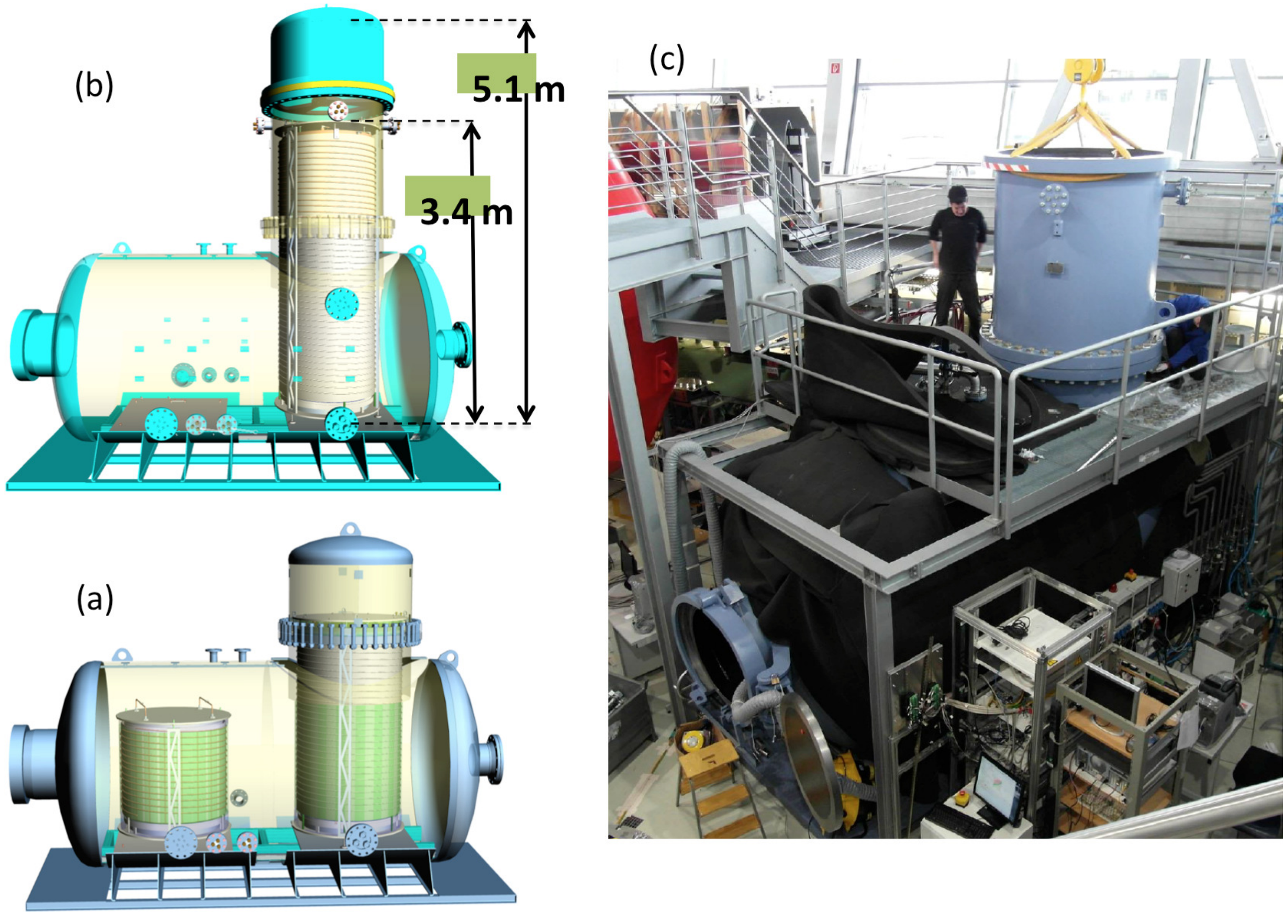

Figure 1. (a): Schematic diagram of the original Uboot. (b): Schematic diagram of the modified Uboot, with its turret extended by 1.7 m. (c): A photograph taken during the Uboot modification.

In order to search for the predicted transition at Ra^*, we built a large facility at the Max-Planck-Institute for Dynamics and Self-Organization (MPIDS) in Göttingen. It consists of a pressure vessel [shown in Fig. 1(a)], now known as the "Uboot of Göttingen", with a volume of about 25 m^3. Attached to and serving the Uboot (as well as an even larger wind tunnel) is a closed gas handling system that could deliver up to 2000 kg of sulfur hexafluoride (SF_6) to the Uboot, thereby filling the system to pressures as high as19 bars. As shown in the figure, the Uboot could contain two cylindrical convection facilities, one each with aspect ratio (diameter/height) $\Gamma = 1.00$ and 0.50, both with sample diameters of 1.1 m. The accessible Ra range was $10^{12} \lesssim Ra \lesssim 1.5 \times 10^{15}$ for $\Gamma = 0.50$ and $2 \times 10^{11} \lesssim Ra \lesssim 2 \times 10^{14}$ for $\Gamma = 1.00$.

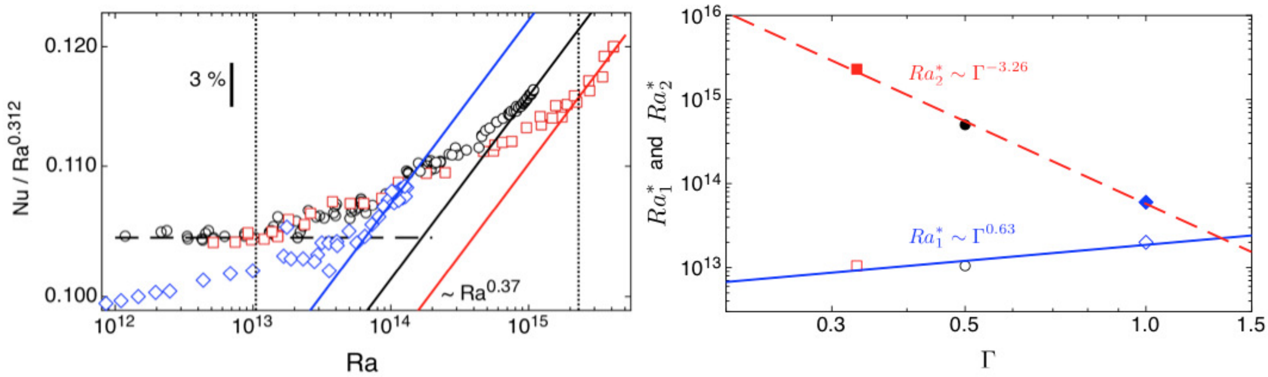

Figure 2. (a): The reduced Nusselt number $Nu/Ra^{0.312}$ as a function of Ra. Diamonds: $\Gamma = 1.00$. Circles: $\Gamma = 0.50$. Squares: $\Gamma = 0.33$. (b): The dependence of the ultimate-state transition-range from Ra_1^* to Ra_2^* on the aspect ratio Γ.

For $\Gamma = 0.5$ it was reported before [6, 2] that the ultimate-state transition was found to occur over a range of Ra, from $Ra_1^* \simeq 10^{13}$ to $Ra_2^* \simeq 5 \times 10^{14}$. For $\Gamma = 1.00$ Ra_1^* was found to be nearly the same for $\Gamma = 1.00$ [5]; but at this conference (Ahlers et al.) we report that $Ra_2^* \simeq 7 \times 10^{13}$ is much lower than it was for $\Gamma = 0.50$.

Recently we modified the Uboot by adding a 1.7 m extension to its turret [Fig. 1(b)]. The modified facility could then accommodate a longer sample, with $\Gamma = 0.33$ (internal height $L = 3.3$ m). It could then also accommodate a rotating system with $\Gamma = 0.50$ as reported at this conference by van Gils et al. Figure 1(c) shows a photograph taken during the modification process which conveys some impression of the physical size of the facility.

In Fig. 2(a) we show some results for the Nusselt number Nu in the reduced form $Nu/Ra^{0.312}$ as a function of Ra for all three Γ. In the classical state ($Ra \lesssim 10^{13}$) $Nu(Ra)$ is the same for $\Gamma = 0.50$ and 0.33, as also found from direct numerical simulation for smaller Ra (E. van der Poel, private communication). All three solid lines are drawn with a slope corresponding to an effective power law $Nu \propto Ra^{\gamma_{eff}}$ with $\gamma_{eff} = 0.37$ as expected [7, 4] for the ultimate state. One sees that the transition to this dependence moves to higher Ra as Γ decreases. The corresponding estimates of Ra_2^* are confirmed also by Reynolds-number measurements not shown in the present abstracts. The results for Ra_1^* and Ra_2^* are collected in Fig 2(b). They suggest that there will be no transition range for $\Gamma \gtrsim 1.4$. For larger Γ we assume that there will be a unique Ra^*; but obviously future measurements will be of great interest.

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