LONG-RANGE ORDERING OF TURBULENT STRESSES IN 2D TURBULENCE

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Abstract We use filter-space techniques to study the geometric alignment of turbulent stresses and strain rates in an experimental quasi-two-dimensional weakly turbulent flow. When these stresses and strains are misaligned, the usual turbulent energy cascade can be suppressed; and more generally, the relative alignment of these two tensors determines the direction of the cascade. We show that as a function of length scale, the turbulent stress undergoes a transition to system-spanning order. However, by exploring analogously defined quantities in a field built from random Fourier modes, we see qualitatively similar behavior, suggesting that at least some of this ordering is purely kinematic. By comparing our results from the experiment and the random field, we highlight the role played by the orientation of the rate of strain tensor in the energy transfer process; additionally, our results allow us to pose several intriguing conjectures.

OVERVIEW

The defining characteristic of turbulent flows is their net propensity to transfer energy between scales in a coherent, directional cascade process. The cascade is typically described in Fourier space, where the notion of "scale" can be made precise; however, by working only in Fourier space, we lose all connection between the energy cascade and the spatial degrees of freedom of the flow. A middle ground between these two representations of the flow dynamics can be found by using so-called filter-space techniques [2, 6]: by an a posteriori removal of the small scales of motion from the velocity field via spectral low-pass filtering, the flux of energy through a given length scale can be spatially resolved.

In this formalism, the transfer of energy $\Pi^{(L)}$ between scales in turbulence can be represented as the inner product of a turbulent stress $\tau_{ij}^{(L)}$ and a filtered rate of strain $s_{ij}^{(L)}$, where the superscript (L) denotes a quantity with all variation on length scales smaller than L suppressed. In two dimensions, this relation can be written as [7]

$$\Pi^{(L)} = -\tau_{ij}^{(L)} s_{ij}^{(L)} = -2\lambda_\tau^{(L)} s_{ij}^{(L)} \cos 2\theta_{s\tau}^{(L)}, \tag{1}$$

where $\lambda_\tau^{(L)}$ and $\lambda_s^{(L)}$ are the largest eigenvalues of $\tau_{ij}^{(L)}$ and $s_{ij}^{(L)}$, respectively, and $\theta_{s\tau}^{(L)}$ is the angle between the corresponding eigenvectors. Since by construction both $\lambda_\tau^{(L)}$ and $\lambda_s^{(L)}$ are non-negative, the sign of the energy flux (and therefore the direction of the cascade) is purely determined by the alignment between the turbulent stress and the large-scale strain.

METHODS

To study the alignment of $\tau_{ij}^{(L)}$ and $s_{ij}^{(L)}$, we analyzed velocity fields measured in a quasi-two-dimensional laboratory flow. To generate nearly two-dimensional flow, we ran a lateral dc electric current through a thin layer (5 mm deep) of electrolytic fluid (16% NaCl by mass in water) that rested over an array of permanent magnets with their dipoles in the vertical direction [4, 5]. To measure the flow fields, we used particle tracking velocimetry, tracking the motion of roughly 35 000 particles per frame at a rate of 60 Hz. By projecting the measured particle velocities onto a basis of streamfunction eigenmodes [4], we ensured the two-dimensionality of our velocity fields. For comparison with the real turbulence dynamics, we also studied "velocity fields" constructed from a set of Fourier modes with random phases, similar to what is done in kinematic simulation [3]. In both cases, to extract the turbulent stresses and filtered strain rates, we convolved the velocity fields with a modified Gaussian kernel that acts as a low-pass filter in Fourier space.

RESULTS

We find that as the filter scale L is increased, the length scale of the spatial orientation fluctuations coarsens, as it must since we are suppressing small-scale variation, as we show in Fig. 1(a-d). But at the same time, the spatial variation of the fluctuations in the orientation of the stress eigenframe coarsens much more rapidly, as shown in Fig. 1(e-h), and we find that at large scales the stress eigenframe is nearly aligned over the entire system.

To quantify the approach to ordering, we use an order parameter ϕ originally developed to study nematic order in two-dimensional liquid crystals [1]. For the strain rate, this order parameter remains small for all L. For the stress, however, it sharply increases at a critical scale L_c and approaches unity nearly as a power law for large L. We also find that the correlation length of the stress orientation fluctuations appears to diverge near L_c. Suggestively, L_c is very close to the energy injection scale in our flow, above which we see net inverse energy transfer (as is expected for two-dimensional

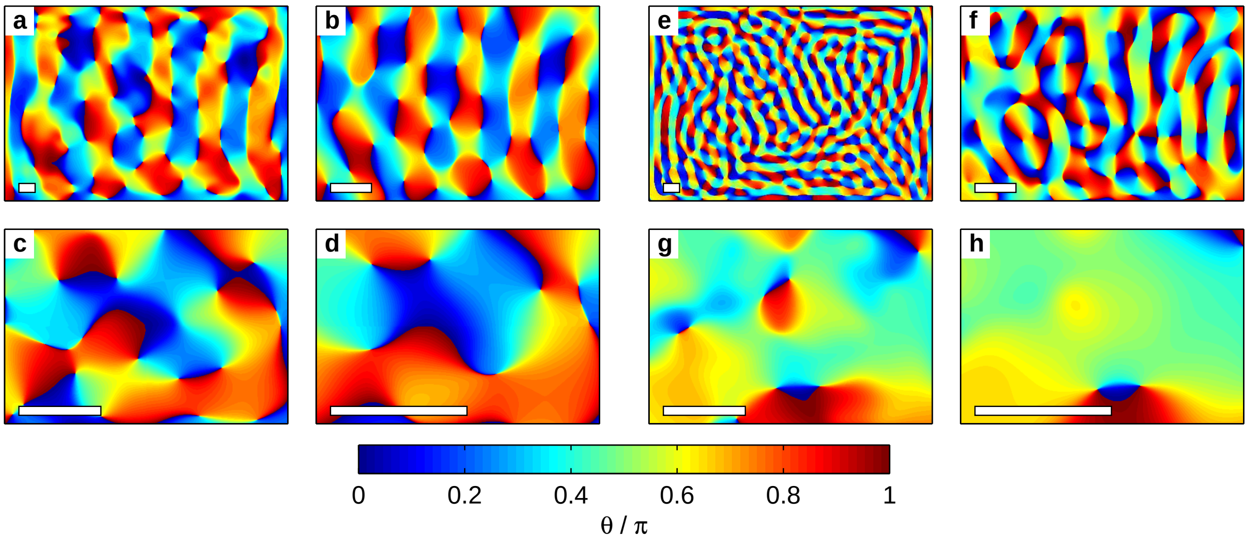

Figure 1. Spatially resolved orientation of the local (a-d) $s_{ij}^{(L)}$ and (e-h) $\tau_{ij}^{(L)}$ eigenframes measured at a single time in the experiment. The color shows the angle between the largest eigenvector and the horizontal axis, in units of π. Data are shown for four filter scales L, shown by the scale bars: (a,e) $0.6L_m$, (b,f) $1.5L_m$, (c,g) $3L_m$, and (d,h) $5L_m$, where L_m is the magnet spacing in the experiment. As L increases, the patterns for both tensors coarsen; but the stress aligns over nearly the entire system at large L, while the strain rate does not.

turbulence). Thus, in the inverse cascade, the turbulent stress displays long-range order, and the approach to ordering shares features with classical critical phenomena.

However, when we measure the same order parameter for the random field, we see very similar effects: the strain rate does not order, but the stress orders perfectly as the filter scale increases. We observe that the ordering transition begins at the length scale of the smallest mode in the system, and the ordering saturates at the largest mode. Thus, at least some of the ordering is purely kinematic. There are, however, differences between the experimental dynamics and the random field; in particular, the degree of ordering fluctuates very strongly in the experiment, while it is nearly static in an ensemble of statistically independent random fields. We will discuss these similarities and differences in detail, and propose some conjectures and directions for future research.

References

[1] P. M. Chaikin and T. C. Lubensky. *Principles of Condensed Matter Physics*. Cambridge University Press, Cambridge, 2000.

[2] G. L. Eyink. Local energy flux and the refined similarity hypothesis. *J. Stat. Phys.*, **78**:335–351, 1995.

[3] J. C. H. Fung and J. C. Vassilicos. Two-particle dispersion in turbulentlike flows. *Phys. Rev. E*, **57**:1677–1690, 1998.

[4] D. H. Kelley and N. T. Ouellette. Onset of three-dimensionality in electromagnetically forced thin-layer flows. *Phys. Fluids*, **23**:045103, 2011.

[5] Y. Liao and N. T. Ouellette. Spatial structure of spectral transport in two-dimensional flow. *J. Fluid Mech.*, **725**:281–298, 2013.

[6] M. K. Rivera, W. B. Daniel, S. Y. Chen, and R. E. Ecke. Energy and enstrophy transfer in decaying two-dimensional turbulence. *Phys. Rev. Lett.*, **90**:104502, 2003.

[7] Z. Xiao, M. Wan, S. Chen, and G. L. Eyink. Physical mechanism the inverse energy cascade of two-dimensional turbulence: a numerical approach. *J. Fluid Mech.*, **619**:1–44, 2008.