NONEQUILIBRIUM AND CLASSICAL DISSIPATION SCALINGS IN DNS OF HOMOGENEOUS ISOTROPIC DECAYING TURBULENCE

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Abstract We present data from direct numerical simulations of homogeneous isotropic decaying turbulence showing both the non-equilibrium and the classical dissipation scalings reported in wind-tunnel experiments of both regular and fractal grid-generated turbulence, i.e. $C_\varepsilon \sim (Re_0/Re_\lambda)^n$ with n of order unity and $C_\varepsilon \sim \mathrm{constant}$, respectively ($Re_0$ and Re_λ are global and local Reynolds numbers). These two dissipation behaviours lead to different power-law decay exponents in both regimes also in accord with the experiments. Finally, we show that in both regimes the maximum non-linear energy cascade flux, Π, reasonably satisfies the classical expectation that $\Pi \sim K^{3/2}/\ell$.

The classical empirical scaling, $\varepsilon \sim C_\varepsilon K^{3/2}/\ell$ (where ε, ℓ, K and C_ε are, respectively, the turbulent kinetic energy dissipation per unit mass, the integral length-scale, the kinetic energy and an empirical constant) is "one of the cornerstone assumptions of turbulence theory" [7, 11]. However, over the past half decade or so, there have been many reports of laboratory experiments on turbulence generated by regular and fractal grids showing regions of the turbulent flow where the classical empirical scaling with $C_\varepsilon \approx \mathrm{constant}$ does not hold and is replaced by $C_\varepsilon \sim (Re_M^{1/2}/Re_\lambda)^n$, with $n \approx 1$ and [5, 10, 6, 2, 3] ($Re_\lambda = \sqrt{K}\lambda/\nu$ and $Re_M = U_\infty M/\nu$ where U_∞ is the inlet velocity, M is an inlet mesh size, $\lambda \equiv \sqrt{10\nu K/\varepsilon}$ is the Taylor microscale and ν is the kinematic viscosity of the fluid). However, in grid-generated turbulence the region exhibiting the nonequilibrium behaviour is typically comprehended between 3 and 20 mesh sizes (for the usual blockage ratios of 30% to 40%), which is closer to the grid than the 'rule-of-thumb' of 30M beyond which the turbulent flow can safely be considered to be homogeneous and fully developed [1]. Perhaps non-surprisingly, much of the skepticism facing the experimental evidence of the nonequilibrium behaviour is precisely the suspition of a non-negligible influence of inhomogeneity and/or inertial range production as well as the possibility of the turbulence not being fully developed [4, 2, 3].

Here we present data from direct numerical simulations of fully-developed decaying homogeneous and isotropic turbulence reproducing the main results of the laboratory experiments without such confounding effects. Contrary to virtually all previous direct numerical simulation of decaying turbulence, the initial condition used for our turbulence decay simulations are velocity fields obtained from fully-developed statistically steady turbulence rather than gaussian noise with a prescribed kinetic energy spectrum. One of the datasets used here (with $N = 512^3$ collocation points and $Re_\lambda \approx 115$ initially) has been presented in Ref. [8] where further details on the simulations and on the numerical methods can be found as well as further physical insight on the mechanisms at play. We complement the previous data with a larger initial Reynolds number dataset ($Re_\lambda \approx 180$) with $N = 1024^3$ collocation points.

The evolution of the normalised energy dissipation against the local Reynolds number for both simulations is shown in Fig. 1a where it can clearly be seen that the dissipation follows two different behaviours. In the first regime from the initial instant to about four turnover times, $C_\varepsilon \sim (Re_0/Re_\lambda)^{1.2}$, and in the second regime, $C_\varepsilon \approx \mathrm{constant}$, until the Reynolds number becomes too small and low Reynolds number effects begin to be felt (the lowest Reynolds numbers for the $N = 512^3$ and $N = 1024^3$ DNSs are $Re_\lambda \approx 45$ and $Re_\lambda \approx 60$, respectively). Our data also indicate that the maximum energy cascade flux, Π, remains approximately proportional to $K^{3/2}/\ell$, although there is a small trend for its numerical value to increase during the initial stages of decay which requires further investigation (Fig. 1a, see caption for

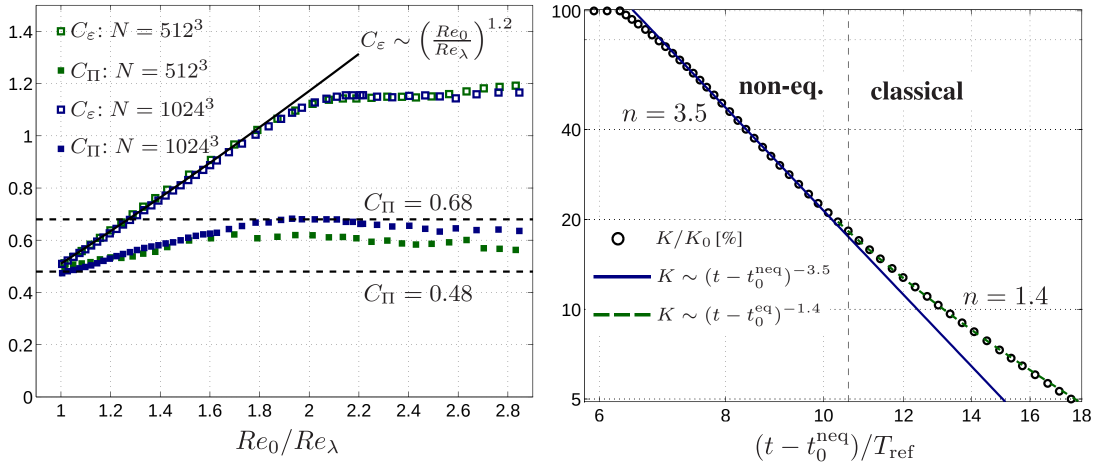

Figure 1. a) Normalised energy dissipation $C_\varepsilon \equiv (3/2)^{5/2}\,\varepsilon\ell/K^{3/2}$ and energy cascade flux $C_\Pi \equiv (3/2)^{5/2}\,\Pi\ell/K^{3/2}$ versus the Reynolds number ratio Re_0/Re_λ. Re_0 is a reference Reynolds number defined as $Re_0 \equiv \sqrt{15}C_\varepsilon^{-2/3}\varepsilon^{1/6}\ell^{2/3}\nu^{-1/2}$ corresponding to Re_λ in a statistically steady state. The factor of $(3/2)^{5/2}$ allows for a direct comparison with experimental surrogates; b) Power-law, $K = A(t+t_0)^{-n}$, regressions for both (——) non-equilibrium regime leading to $t_0/T_{\mathrm{ref}} = 6$ and $n = 3.5$ and (– –) classical regime leading to $t_0/T_{\mathrm{ref}} = 0.7$ and $n = 1.4$. K_0 and $T_{\mathrm{ref}} = \ell_0/\sqrt{K_0}$ are the initial kinetic energy and turnover time, respectively and t_0^{neq} is the virtual time origin for the non-equilibrium power-law fit. The data are fitted with the non-linear method discussed in Ref. [9]. The exponents vary slightly with the data range, however, the exponents in the non-equilibrium period are consistently larger ($n > 2.5$) than in the equilibrium period ($n < 1.6$).

definition of C_ε and Re_0).

The present data also confirms that the behaviour of the dissipation is the main cause for the steeper decay exponents in the nonequilibrium regime. The conservation of a large-scale invariant together with $C_\varepsilon \sim (Re_0/Re_\lambda)^\alpha$ uniquely determines the power-law decay exponent [9]. Based on our C_ε data, together with the range of possible large-scale invariants [9], leads to $1.7 \le n \le 4$ for $\alpha = 1.2$ and $6/5 \le n \le 10/7$ for $\alpha = 0$, which is consistent with the independently fitted power-laws (Fig. 1b).

References

[1] S. Corrsin. *Handbook der Physik*, **8**, pages 524–590. Springer, New York, 1963.

[2] R. J. Hearst and P. Lavoie. Decay of turbulence generated by a square-fractal-element grid. *J. Fluid Mech.*, **741**:567 – 584, 2014.

[3] J. C. Isaza, R. Salazar, and Z. Warhaft. On grid-generated turbulence in the near- and far field regions. *J. Fluid Mech.*, **753**:402 – 426, 2014.

[4] P.-Å. Krogstad and P. A. Davidson. Near-field investigation of turbulence produced by multi-scale grids. *Phys. Fluids*, **24**:035103, 2012.

[5] N. Mazellier and J. C. Vassilicos. Turbulence without Richardson-Kolmogorov cascade. *Phys. Fluids*, **22**:075101, 2010.

[6] K. Nagata, Y. Sakai, T. Inaba, H. Suzuki, O. Terashima, and H. Suzuki. Turbulence structure and turbulence kinetic energy transport in multiscale/fractal-generated turbulence. *Phys. Fluids*, **25**(065102), 2013.

[7] H. Tennekes and J. L. Lumley. *A First Course in Turbulence*. MIT Press, Cambridge, New York, 1972.

[8] P. C. Valente, R. Onishi, and C.B. da Silva. The origin of the imbalance between energy cascade and dissipation in turbulence. *Phys. Rev. E*, **90**(023003), 2014.

[9] P. C. Valente and J. C. Vassilicos. Dependence of decaying homogeneous isotropic turbulence on inflow conditions. *Phys. Lett. A*, **376**:510–514, 2011.

[10] P. C. Valente and J. C. Vassilicos. Universal dissipation scaling for nonequilibrium turbulence. *Phys. Rev. Lett.*, **108**(214503), 2012.

[11] J. C. Vassilicos. Dissipation in turbulent flows. *Annu. Rev. Fluid Mech.*, **47**:95 – 114, 2015.