NUMERICAL INVESTIGATION OF LOCALIZED EXACT SOLUTIONS OF THE NAVIER-STOKES EQUATIONS IN PIPE FLOW

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Abstract The edge state solution in pipe flow at Re=2200 is calculated numerically. The solution has the form of spatially localized puff-like structure drifting downstream. In the moving frame it is represented by a steady average flow and time-periodic pulsation flow. It is shown, that the Kelvin-Helmholtz instability mechanism is not valid for pulsation generation in the edge state flow.

INTRODUCTION

Turbulence in pipe flow at low Reynolds numbers first appears in the form of spatially localized structures, called puffs. Puffs drift downstream with about mean flow velocity. Their streamwise extension of about 10–20 pipe diameters stays almost constant. But puffs are liable to spontaneous diffusion and division. According to [1] at Re>2040 the tendency of puffs division dominates the tendency of diffusion, thus Re=2040 may be considered as a point of statistical phase transition from laminar to turbulent flow in a pipe. An attempt to clarify the mechanism of puff was made in [2]. Although the main statements made in [2] concerning puff self-maintenance look plausible, the number of questions still remain unclear. Direct investigation of turbulent puffs is attended with certain difficulties because of complexity in their behaviour due to stochasticity. In our opinion, certain progress in understanding of puff dynamics can be extracted from investigating more simple structures approximating puffs. As such an approximation we use spatially localized relatively periodic unstable solutions found recently in [3].

RESULTS

We solve incompressible Navier-Stokes equations at Re=2200 for a pipe-flow geometry using finite-difference algorithm of [4]. The computational domain (streamwise period) of $60D$ (D is pipe diameter) is long enough to represent localized puff structure. Following [3] additional constrains of a π-rotational symmetry with respect to the pipe axis and the reflectional symmetry of solution are applied. Starting with some 3D perturbation of Poiseuille flow numerical solution evolves either back to laminar Poiseuille state or towards localized puff depending on the amplitude of perturbation. Variation of initial condition by a bisection method is used to find dividing edge [5]. After an initial transient the temporal evolution relaxes onto a spatially localized puff-like solution drifting with a speed $c = 1.38U$ (U is bulk velocity) and subjected with periodic oscillations with period $T = 15.0D/U$.

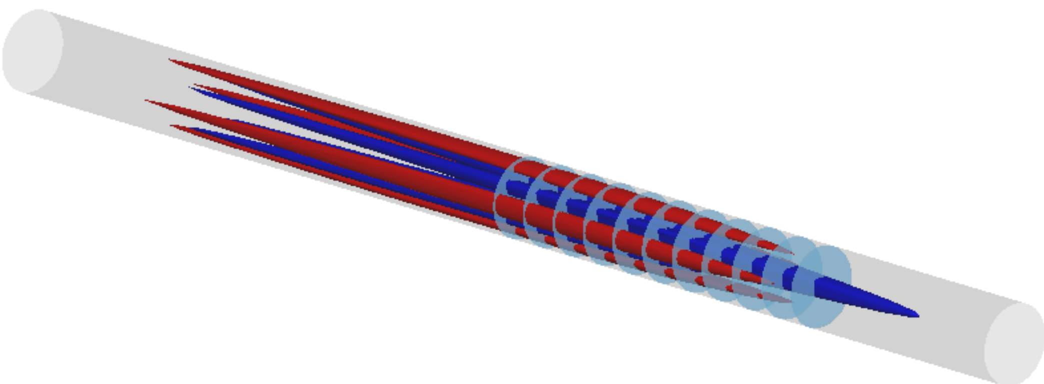

Figure 1. Time-average edge state at Re=2200. Isosurfaces of difference between streamwise velocity and laminar flow (red for positive, blue for negative). Flow direction is from left ro right. Cyan cross-sections indicate locations of maps in figure 2.

In a moving frame of reference edge-state solution can be represented as a $\pi/2$-rotationally symmetric average flow and a π-symmetric pulsating time-periodic flow traveling downstream. Average flow is characterized by velocity deficit on the pipe axis and a set of 4 pairs of low- and high-velocity streaks elongated along the pipe wall. The averaged streamwise velocity field of edge state $\overline{u}$ is visualized in the figure 1.

Pulsating counterpart of the flow ($u' = u - \overline{u}$) represents a wave, that arises in the center area of the puff and travels downstream with a speed of about $0.2U$. Distribution of pulsating energy is shown in the figure 2 (top). The main concern

of the current work is the mechanism whereby average flow produces pulsations. According to [2] pulsations in a puff originate in the thin wall layers formed by low-speed streaks through the Kelvin-Helmholtz instability mechanism. Our analysis shows that this mechanism is not valid for the edge-state flow. The maps of total pulsation energy generation (energy production $-u'(u',\nabla)\overline{u}$ minus dissipation $2\nu\epsilon_{i,j}\epsilon_{i,j}$ where ϵ is a rate of strain tensor) averaged over time period are shown in figure 2 (bottom). In contrast to mechanism of [2] pulsations are supported mostly in the near-axis area between high and low velocity streaks. No visible pulsation energy production is found in the area of low velocity streaks.
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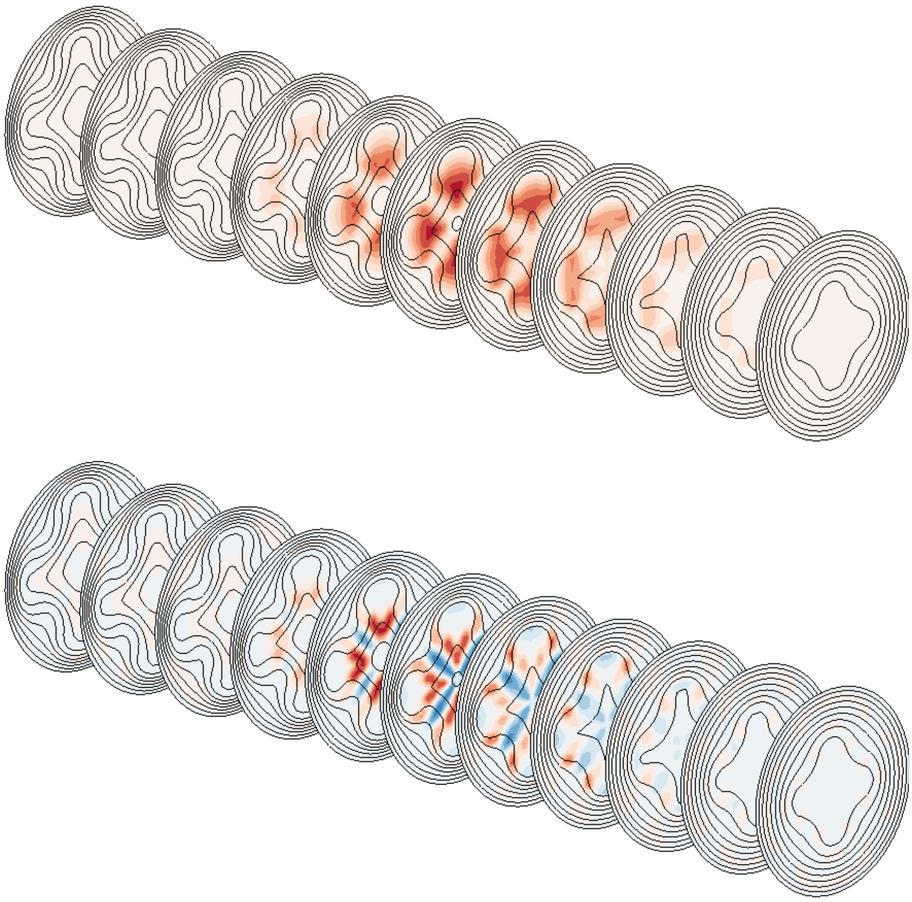

Figure 2. Time-average pulsation energy (top) and time-average pulsation energy generation (production minus dissipation, bottom). Colors: positive is red, negative is blue. Location of the cross-sections is shown in the figure 1.

References

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