THREE-DIMENSIONAL STRUCTURE OF QUANTIZED VORTICES IN ROTATING BOSE-EINSTEIN CONDENSATES

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Abstract Bose-Einstein condensates (BEC) are ideal superfluid systems to realize quantum turbulence (QT): vortex cores in BECs are larger than in superfluid Helium, making easier their observation. Recent experimental and numerical studies [8] reported that vortex states in BEC can evolve towards a turbulent regime when an oscillatory excitation is applied. We discuss in this work how to accurately prepare initial states with vortices before running numerical simulations of QT based on the Gross-Pitaevskii equation. The case of a dense Abrikosov lattice in a fast rotating BEC is presented. High resolution numerical simulations using parallel computing are used to accurately capture physically important features of the vortices (vortex radius, inter-vortex spacing, vortex density profile).

INTRODUCTION

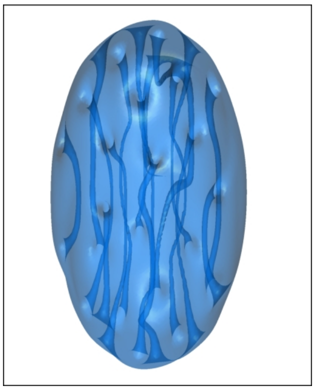

Figure 1. Example of 3D simulation of an Abrikosov type vortex lattice with tangled vortices in rotating BEC.

Superfluid systems with a large number of quantized vortices tangled in space can evolve to quantum turbulence (QT) [3, 6]. There are several practical reasons for which the BEC is a perfect superfluid system to realize and study QT: vortex cores are larger than in superfluid Helium (and thus more easily observable); the high controllability of the system and its finite size (controlled by the trapping potential) make possible new scenarios for the transition to QT. Recent experimental and theoretical studies [7, 8] reported different possible routes to QT in BEC. One of the explored paths was the use of an oscillatory excitation [8]: for a certain range of the amplitude of the excitation, an increasing number of vortices was generated, and the vortex lattice finally evolved into a turbulent regime.
The present contribution is concerned with the numerical investigation of the evolution of a dense Abrikosov vortex lattice (see figure 1) towards QT. We focus here on the generation of different initial conditions of rotating BEC with vortices. As in classical fluid turbulence, the numerical and physical accuracy of the initial condition could be crucial in computing properties of numerically generated QT. To accurately simulate configurations with a large number of vortices, a new numerical system solving the Gross-Pitevskii equation was designed and implemented using MPI-OpenMP parallel programming.

NUMERICAL SYSTEM

We consider a pure BEC of N atoms confined in a trapping potential $\mathbf{V}$ rotating with angular velocity $\mathbf{\Omega}$. The evolution of the system is described by the Gross-Pitaevskii equation (GPE):

$$i\frac{\partial}{\partial t}\psi(\mathbf{x},t) = \left(-\frac{1}{2}\nabla^2 + \mathbf{V}(\mathbf{x}) + \beta|\psi(\mathbf{x},t)|^2 - \mathbf{\Omega}\cdot\mathbf{L}\right)\psi(\mathbf{x},t), \quad \text{in } \mathcal{D}\subset\mathbb{R}^3, \tag{1}$$

where β is the nonlinear interaction coefficient (all variables are dimensionless). The wave function ψ is normalized to unity, *i. e.* $\int_{\mathcal{D}}|\psi|^2 = 1$. The angular momentum is defined as $\mathbf{L} = \mathbf{x}\times\mathbf{p}$, with $\mathbf{p} = -i\nabla$ the linear momentum. We simulate the dynamics of the BEC starting from an initial condition representing stable or meta-stable states of the system. Such steady states are sought as solutions of the form $\psi(\mathbf{x},t) = u(\mathbf{x})\exp(-\mu t)$, with μ the chemical potential. Consequently, we also have to solve the equation for $u(\mathbf{x})$, obtained directly from (1) as a nonlinear eigenvalue problem, also called the stationary GPE; we use a pseudo-time (or imaginary-time, τ) propagation formulation:

$$\frac{\partial}{\partial\tau}u(\mathbf{x},\tau) = \left(\frac{1}{2}\nabla^2 - \mathbf{V}(\mathbf{x}) - \beta|u(\mathbf{x},\tau)|^2 + \mathbf{\Omega}\cdot\mathbf{L}\right)u(\mathbf{x},\tau), \quad \text{in } \mathcal{D}\subset\mathbb{R}^3. \tag{2}$$

We developed a new parallel MPI-OpenMP numerical code that is able to solve both real-time (1) and imaginary-time (2) Gross-Pitaevskii equations. For the space discretization we use either Fourier pseudo-spectral methods (with periodic boundary conditions) or compact sixth order finite difference schemes (with periodic or homogeneous Dirichlet boundary conditions). Different time integration schemes are used for the two equations: for (1) we have the choice between Strang splitting, Crank-Nicolson and relaxation methods (see [1] for a review); for (2) we use a semi-implicit Euler or Crank-Nicolson scheme (see [2] for a review). The numerical code performed with very good scaling properties on parallel supercomputers up to 16,000 cores. The highest grid resolution used in computations was 1024^3.

RESULTS AND DISCUSSION

One way to obtain a very dense vortex lattice in a BEC is to consider a fast rotating condensate with large nonlinear interaction constant β. We illustrate in figure 2 the case of a condensate trapped in a hamonic+quartic potential which allows very large rotation rates. The physical parameters correspond to experiments by [4]; some of these configurations were simulated in [5]. We focus in this part on the validation of the computed stationary state by extracting physically relevant properties from numerical data. Figure 2 shows very good agreement between numerical data and the theory by [9] for the vortex radius r_v (proportional to the healing length ξ) and the inter-vortex spacing b_v (parameter of the hexagonal lattice). It is also interesting to note that density profiles of vortices $\rho(r)$ are different from classical approximations: Thomas-Fermi (TF) $\rho(r) \sim (r/\sqrt{2\xi^2 + r^2})^2$ and Lowest-Landau-Level (LLL) $\rho(r) \sim exp[-r^2/2l^2])^2$, $l = (\sqrt{3}/2\pi)^{1/2} b_v$. This indicates that an initial condition with artificially planted vortices using these theories might be inaccurate for starting numerical simulations of QT. More dense lattices (up to 300 vortices) were obtained for large values of the rotation rate Ω and interaction constant β. In the next step of this study, steady vortex lattices (as illustrated in figure 2) will be used as initial condition in real-time GPE (1) and submitted to external oscillatory excitation, as in [8], to explore routes to QT. This part is currently under investigation.

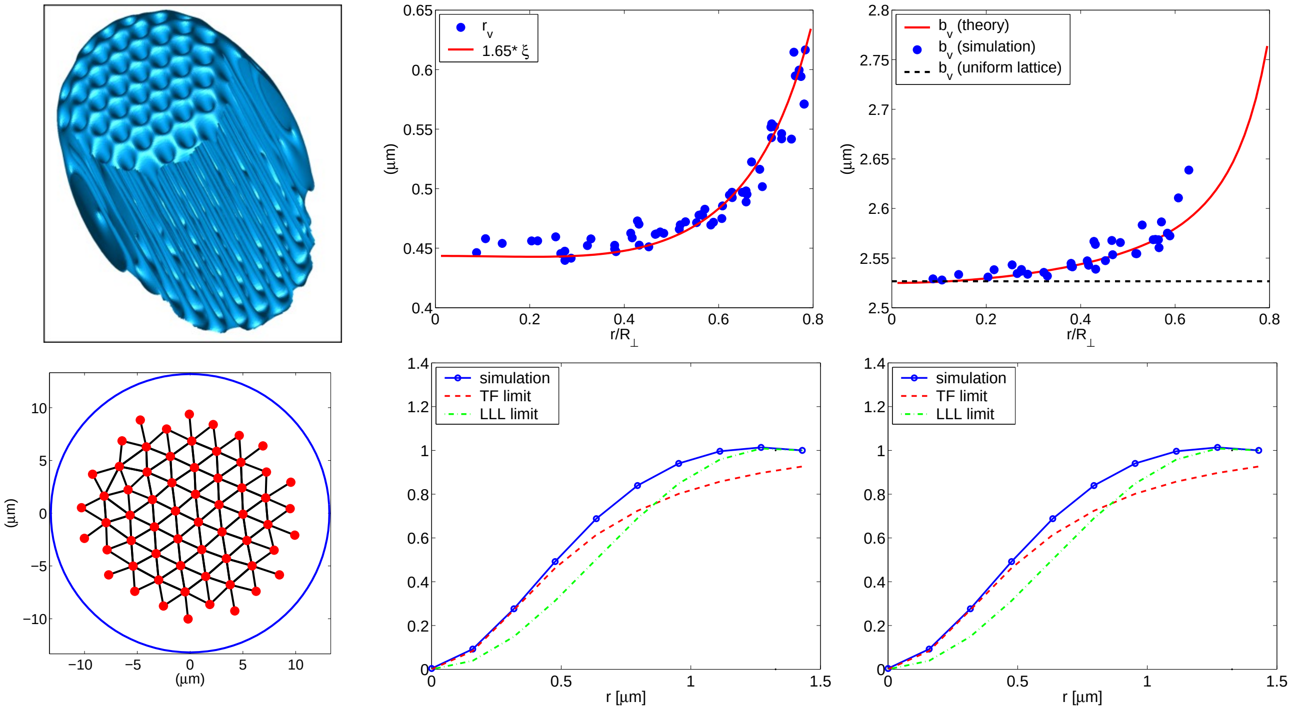

Figure 2. Numerical simulation of a fast rotating BEC with a harmonic+quartic trapping potential. Three-dimensional structure of the Abrikosov vortex lattice (in blue) and its representation in the mid-plane (red dots). Computed characteristics of the vortex lattice and comparison with theory: vortex radius r_v, inter-vortex spacing b_v (upper range); density profiles $\rho(r)$ for two vortices (lower range).

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