WAVE EXCITATIONS IN ADJACENT VORTEX FILAMENTS

Niklas Hietala[1], Risto Hänninen [1], Hayder Salman [2] & Carlo F. Barenghi [3]
[1]*Low Temperature Laboratory, O.V. Lounasmaa laboratory, Aalto University, FI-00076 AALTO, Finland*
[2]*School of Mathematics, University of East Anglia, Norwich Research Park, Norwich NR4 7TJ, United Kingdom*
[3]*Joint Quantum Centre Durham-Newcastle, School of Mathematics and Statistics, Newcastle University, Newcastle upon Tyne NE1 7RU, United Kingdom*

Abstract The interactions of the nearest neighbour vortices are argued to play a significant role in the crossover range of scales that lies between the Kolmogorov-Richardson cascade and the Kelvin wave driven cascade in superfluid turbulence. In this work, we study how a wave excitation (a Kelvin wave or a soliton) on a vortex affects a nearby straight vortex. Our numerical simulations reveal that coherent excitations can hop from one vortex filament to another whilst retaining their coherent properties.

INTRODUCTION

Turbulence in superfluids takes the form of a quasiclassical Kolmogorov cascade at length scales larger than the intervortex separation. At scales less than the intervortex separation, energy transfer to decreasing length scales can be sustained by a Kelvin wave cascade that can exist on individual vortex lines. What happens at the crossover of these two regimes is more or less an open question. Kozik and Svistunov proposed a scenario with a chain of cascades driven by three different mechanisms [1]. In one of these subregimes, interactions between nearest-neighbour vortex lines have an important role.
In this work, we perform numerical simulations for parallel vortices. We use the vortex filament model, where the vortices are described as discretized space curves $\mathbf{s}(t)$ (t is the time) and the vortex points move according to the Biot–Savart law

$$\mathbf{v}_{\mathrm{s}} = \frac{\kappa}{4\pi}\hat{\mathbf{s}}' \times \mathbf{s}'' \ln\left(\frac{2\sqrt{l_+ l_-}}{e^{1/2} a_0}\right) + \frac{\kappa}{4\pi}\int \frac{(\mathbf{s}_1 - \mathbf{s}) \times d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{s}|^3} . \quad (1)$$

The singularity at a vortex point is removed by isolating the local contribution that is contained in the first term. Lengths of the two adjacent line segments connected to $\mathbf{s}$ are denoted by $l_\pm$. These line segments are excluded from the Biot-Savart integral contained in the second term. Prime denotes differentiation with respect to arc length. Here we have the quantum of circulation $\kappa = h/m$ and the core diameter a_0. For helium-4 $\kappa = 0.0997$ mm^2/s and $a_0 \approx 10^{-7}$ mm. Since the core radius is several orders of magnitude smaller than any other length scale, it is reasonable to model superfluid vortices as vortex filaments. Our simulations are performed with periodic boundary conditions in the direction of the vortex and assuming zero temperature such that there is no mutual friction acting on the vortices.

KELVIN WAVES

The simplest case to study is two coaxial vortices, one straight and one with a Kelvin wave. Although this is a somewhat unphysical situation, where the straight vortex is trapped within the helical vortex, the advantage in considering this setup is that both of the vortices have the same axis of symmetry. Subsequent integration of the Eq. (1) reveals that a Kelvin wave with the same wave number as that initially present on the perturbed vortex will grow on the initially straight vortex. This is compensated by a decrease in the amplitude of the initially perturbed vortex until it becomes straight. The cycle then continues resulting in a recurrence as can be seen in Fig. 1.
A similar kind of recurrence is also seen when the helical vortex is not wound around the straight vortex but is placed next to it. The difference is that if the vortices do not have the same symmetry axis, additional modes will appear (multiples of the initial mode). The initial distance between the vortices also affects the period of the recurrence. The phenomena can be generalised in situations with more than two vortices. The ensuing behaviour of the vortices becomes more complex.
Our simulations show that a breather is likely to occur for a large amplitude Kelvin wave. A breather is a nonlinear wave in which energy concentrates in a localized and oscillatory manner. In the case of a vortex filament, a breather is manifested as a looplike excitation of the helical vortex [2]. Large amplitude excitations of this kind often result in reconnections between adjacent vortices. Reconnections between neighbouring vortices and possible self-reconnections may lead to the generation of small vortex loops.

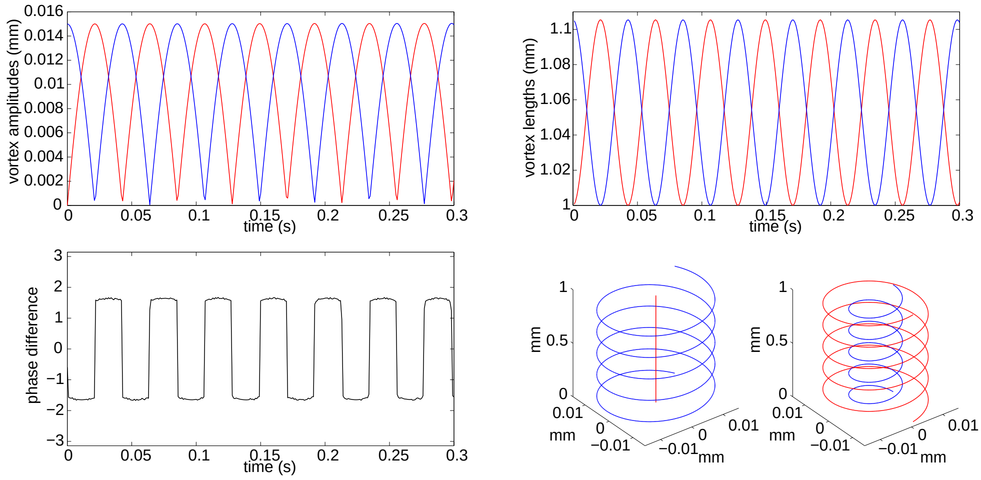

Figure 1. The initial configuration was a helical vortex (blue) wound around a straigth vortex (red). Kelvin wave amplitude was 0.015 mm, mode 5 and length of the z-period is 1 mm. Vortex amplitudes, lengths and the phase difference as functions of time. The jump in the phase occurs, when one of the vortices straightens. Vortex configurations at $t = 0$ s and $t = 0.015$ s.

SOLITONS

An example of localized wave excitation is the Hasimoto soliton [3]. The soliton solution is

$$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ s \end{pmatrix} + \frac{2\mu/\nu}{\cosh\eta} \begin{pmatrix} \cos\theta \\ \sin\theta \\ -\sinh\eta \end{pmatrix} \tag{2}$$

where $\mu = \nu^2/(\nu^2 + \tau_0^2)$, $\eta = \nu(s - 2\tau_0 t)$ and $\theta = \tau_0 s + (\nu^2 - \tau_0^2)t$. The soliton is parametrized with two parameters: τ_0, torsion, and ν, half of the maximum curvature. In our simulations we have typically used $s \in [-10, 10]$. We rescaled all the coordinates so that $z \in [0, 1]$ in mm's. While this scaling changes the actual values of the torsion and the maximum curvature, it does not affect the ratio of the two parameters, τ_0/ν, which is the key parameter in classifying the properties of a soliton.

When $\tau_0/\nu \gg 1$ we observe something similar to the recurrence with a Kelvin wave. A soliton-like excitation appears in the adjacent vortex, and the amplitudes of the excitations change periodically (see Fig. 2). The amplitudes decrease in time, which tells us that some of the energy is dispersed. In the figure showing the vortex configuration, some additional waves are clearly visible.

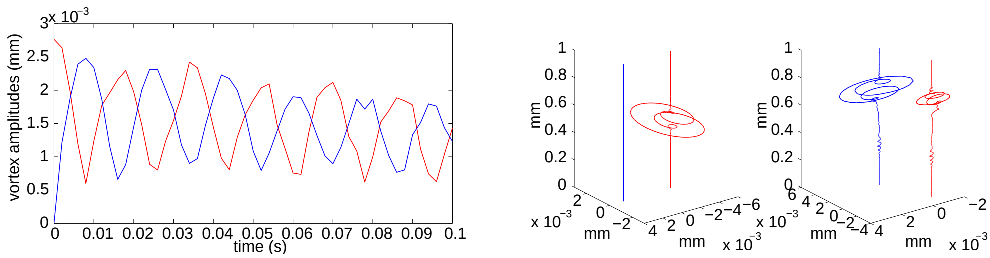

Figure 2. Amplitudes of the vortices as functions of time and the vortex configurations at $t = 0$ s and $t = 0.006$ s. The blue vortex was initially straight and the red vortex had a soliton with $\tau_0/\nu = 10/3$. Distance between the vortices was 0.005 mm and the z-period is 1 mm.

References

[1] E. Kozik and B. Svistunov. Kolmogorov and Kelvin-wave cascades of superfluid turbulence at $T = 0$: What lies between. *Phys. Rev. B* **77**: 060502, 2008.

[2] H. Salman. Breathers on quantized superfluid vortices. *Phys. Rev. Lett.* **111**: 165301, 2013.

[3] H. Hasimoto. A soliton on a vortex filament. *J. Fluid Mech.* **51**: 477–485, 1972.