WALL TO WALL OPTIMAL TRANSPORT

Charles R. Doering[1,2,3], Andre N. Souza[2], Gregory P. Chini[4] & Pedram Hassanzadeh[5]
[1]*Department of Physics, University of Michigan*
[2]*Department of Mathematics, University of Michigan*
[3]*Center for the Study of Complex Systems, University of Michigan*
[4]*Department of Mechanical Engineering, Program in Integrated Applied Mathematics and Center for Fluid Physics, University of New Hampshire*
[5]*Centre for the Environment and Department of Earth and Planetary Sciences, Harvard University*

Abstract The calculus of variations is employed to find steady divergence-free velocity fields that maximize transport of a tracer between two parallel walls held at fixed concentration for one of two constraints on flow strength: a fixed value of the kinetic energy or a fixed value of the enstrophy (the mean square rate of strain in this situation). The optimizing flows realize upper limits on convective transport in this scenario. We interpret the results in the context of buoyancy-driven Rayleigh–Bénard convection problems that satisfy the flow intensity constraints, enabling us to investigate how optimal transport scalings compare with upper bounds on *Nu* expressed as a function of the Rayleigh number *Ra*.

PROBLEM, APPROACH, AND RESULTS

The goal of this work is to help understand heat transport high Rayleigh number convection by producing and characterizing flows that optimize scalar transport for comparison with naturally occurring turbulent flows. In [1] Hassanzadeh *et al* introduced a new to the continuing challenge to derive sharp bounds on heat transport in Rayleigh-Bénard convection. The particular conditions and constraints considered are that the flow is confined between two impermeable parallel boundaries and endowed with a specified root-mean-square speed or vorticity, and that the concentration of a scalar tracer is constant on each boundary, i.e. the boundaries are isothermal. For simplicity we restrict attention to steady flows in two spatial dimensions. We do not require the incompressible velocity field to satisfy any familiar momentum equation (e.g. for motion driven by the buoyancy force) but only a suitable bulk amplitude constraint which, after the fact, may be interpreted in terms of well-known dissipation–transport balances (e.g. for classical Rayleigh-Bénard convection). Unlike prior approaches to deriving rigorous bounds, however, in this analysis the full advection-diffusion equation for the transported scalar field is enforced as a constraint.

References

[1] P. Hassanzadeh, G. P. Chini and C. R. Doering. Wall to wall optimal transport. *Journal of Fluid Mechanics* **751**: 627–662, 2014.