REVISITING THE PARADIGM OF PREDOMIMANT VORTEX STRETCHING AND RELATED

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Abstract We address two issues of paradigmatic nature. The first one, among other things, concerns the fact how it is possible to have predominant vortex stretching mostly due to the largest eigen-strain without predominant alignment of vorticity $\boldsymbol{\omega}$ with the eigen-vector $\boldsymbol{\lambda}_1$ corresponding to the largest eigen-strain Λ_1. The second is about the paradigmatic qualitative differences between the fields of vorticity and that of rate of strain.

ENSTROPHY PRIODUCTION VERSUS VORTICITY-STRAIN ALIGNMENS

The paradigm is due to Taylor [1, 2], motivated by the assumption of von Karman [3] that $\langle\omega_i\omega_j s_{ij}\rangle \equiv 0$. Taylor [2] provided experimental evidence in favor of positiveness of enstrophy production, though his reasoning was based on the incorrect analogy with material lines: *Turbulent motion is found to be diffusive, so that particles which were originally neighbors move apart as motion proceeds. In a diffusive motion the average value of d^2/d_0^2 continually increases. It will be seen therefore . . . that the average value of ω^2/ω_0^2 continually increases* [2]. We note here that the evolution of material lines is of purely kinematic nature, whereas the underlying cause of vortex stretching in turbulent flows is a *dynamical* one. We discuss this issue in more detail in the presentaion. This above mentioned analogy, in turn, produced an "intuitive" expectation of the statistical predominance of alignment between vorticity $\boldsymbol{\omega}$ and the largest eigen-vector of the rate of strain tensor $\boldsymbol{\lambda}_1$ (the eigenvalues of s_{ij} are ordered as $\Lambda_1 > \Lambda_2 > \Lambda_3$). However, massive evidence for a broad range of Reynolds numbers, including $Re_\lambda \approx 10^4$, see e.g. references in [4], pointed unequivocally to predominant alignment between vorticity $\boldsymbol{\omega}$ and the intermediate rate of eigen-vector $\boldsymbol{\lambda}_2$ of the rate of strain tensor, and was considered as a puzzle with attempts to discover the "right alignment", e.g. [5], [6]. In reality, it appears that the predominant vortex stretching is indeed due to alignment $\boldsymbol{\omega},\boldsymbol{\lambda}_1$, but for this there is no need for the statistical predominance of this alignment as massively expected. This is because statistical dominance is not synonymous to dynamical relevance. We review and bring new evidence how this can happen resolving this apparent “contradiction”. Among the reasons is that unlike the strain self-production (see below) the interaction of vorticity and strain involve important issues of geometrical nature which are complicated by the nonlocal relation between them. For example, $\omega_i\omega_j s_{ij} = \omega^2\Lambda_k \cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_k)$, k=1,2,3, so that the enstrophy production is essentially dependent on (i) the magnitude of ω^2, (ii) the eigenvalues Λ_i of the rate of strain tensor s_{ij}, (iii) the alignments between vorticity $\boldsymbol{\omega}$ and the eigenframe $\boldsymbol{\lambda}_k$ of the rate of strain tensor s_{ij} and (iv) correlations between the three (i) – (iii). We demonstrate that the main contribution to the enstrophy production and its rate is indeed due to the first term associated with the $\boldsymbol{\omega},\boldsymbol{\lambda}_1$ alignment, see [4] and references therein. For example, in the field experiments with $Re_\lambda \approx 10^4$ [4, 7] the relation between the mean of the three contributions $\omega^2\Lambda_1 \cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_1)$: $\omega^2\Lambda_2 \cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_2)$: $\omega^2\Lambda_3 \cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_3) = 3.1 : 1.0 : -2.1$. The dynamical dominance of the term associated with the $\boldsymbol{\omega},\boldsymbol{\lambda}_1$ alignment is much stronger for the corresponding rates, i.e. $\omega_i\omega_j s_{ij}/\omega^2 = \Lambda_k \cos_2(\boldsymbol{\omega},\boldsymbol{\lambda}_k)$; $\Lambda_1 \cos2(\boldsymbol{\omega},\boldsymbol{\lambda}_1)$: $\Lambda_2 \cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_2)$: $\Lambda_3 \cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_3) = 4.9 : 1.0 : -3.8$, which exhibits far stronger role of strain and $\boldsymbol{\omega},\boldsymbol{\lambda}_i$ alignments. For PDFs of involved quantities see Fig.1.

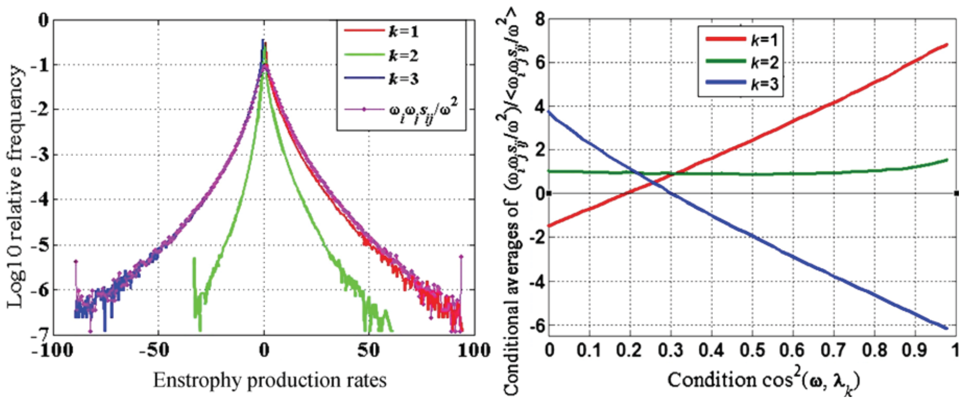

Figure. 1. Left: Histograms of the total rate of enstrophy production $\omega_i\omega_j s_{ij}/\omega_2$ and separate contributions $\Lambda_k \cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_k)$, k = 1, 2, 3. It seen clearly that the main contribution to the total on the positive part comes from $\Lambda_1 \cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_1)$. Right: Conditional averages of EPR = $\omega_i\omega_j s_{ij}/\omega_2$ on $\cos^2(\boldsymbol{\omega},\boldsymbol{\lambda}_k)$. We preferred to use the evidence as obtained for real physical fields to avoid any abuse by some additional processing such as decompositions, etc.

VORTICITY VERSUS STRAIN - PARADIGMATIC DIFFERENCES

There are two concomitant qualitatively universal physical mechanisms turning turbulence into a strongly dissipative and rotational phenomenon. These are the predominant production of the rate of strain tensor, s_{ij} and vorticity, ω_i . The rate of strain tensor, s_{ij} and vorticity, ω_i are just the symmetric and antisymmetric parts of the tensor of velocity derivatives $A_{ij} = \partial u_i/\partial x_j \equiv s_{ij} + 1/2\ \varepsilon_{ijk}\omega_k$. Though formally the two representations are trivially equivalent, it is also trivially obvious that it is more appropriate from the physical point of view to use strain and vorticity as corresponding to the two fundamental properties of turbulence as strongly dissipative and essentially rotational due to the nonlinearity of the NSE [8]. We remind that the direct causal relation to dissipation is not the only role played by strain $\varepsilon = 2\nu s_{ij}s_{ij}$ in turbulent flows, rather than to enstrophy.

There is a conceptual and qualitative difference between the nonlinear interaction between vorticity and strain, e.g. $\omega_i\omega_j s_{ij}$ and the self-amplification of the field of strain, $-s_{ij}s_{jk}s_{ki}$. The latter is a specific feature of the dynamics of turbulence having no counterpart (more precisely analogous—not more) in the behavior of passive and also active objects. This process, i.e., $-s_{ij}s_{jk}s_{ki}$ is local in contrast to $\omega_i\omega_j s_{ij}$, as the field of vorticity and strain are related nonlocally.

In contrast to the common view: *It seems that the stretching of vortex filaments must be regarded as the principal mechanical cause of the high rate of dissipation which is associated with turbulent motion* [2] it is the production of strain which is responsible both for

(i) the enhanced dissipation of turbulence and in particular, for what is called "cascade" as resulting in enhanced dissipation, which is not surprising as the appropriate level of dissipation moderating the growth of turbulent energy is achieved by the build up of strain of sufficient magnitude, and

(ii) the enstrophy production either. In other words, apart of dissipation the strain field plays the role (among several others) of an engine producing the whole field of velocity derivatives, both itself and the vorticity, with compression aiding the prevalent production of strain and stretching aiding the prevalent production of enstrophy.

It is of special importance on paradigmatic level that it is the strain production which is responsible for the finite overall dissipation at (presumably) any however large Reynolds numbers.

The fascinating aspect of the above non-conformistic statements is that they become literally obvious when one takes the labor to look at both equations, i.e. for ω^2 and for s^2 too. An important a bit subtler aspect is that the field of strain is efficient in the above two missions only with the aid of vorticity, i.e. only if the flow is rotational, since otherwise the strain (self-)production, $-s_{ij}s_{jk}s_{ki}$, for an irrotational flow field is just a divergence, $s_{ij}s_{jk}s_{ki} = \partial\{\cdot\cdot\cdot\}/\partial x_i$ [9].

Among other differences of special interest is that all key nonlinearities appear to be much stronger in the strain dominated regions rather than in regions with concentrated vorticity, in contrast to the common expectation that, for example, the vorticity amplification process will be strongest where the vorticity already happens to be large.

A similar phenomenon of strain dominance is observed in wall bounded turbulent flows as concerns the Reynolds stress and production of turbulent kinetic energy [4].

If time permits other differences will be reviewed.

The bottom line here is that on the paradigmatic level it is the nonlinearity that is responsible for such most basic key properties of turbulence as essentially rotational and strongly dissipative phenomenon. This is just because the excitation of small scales is due to the nonlinearity of the NSE.

It is worth of emphasizing that these two concomitant key properties and processes are observed in a rather straightforward manner, so that there is no need for "cascades", decompositions etc. These latter so far are mainly obscuring rather than helping to "understand the physics of cascades" or whatever, not to say about the positively skewed nature of the $-s_{ij}s_{jk}s_{ki}$ and $\omega_i\omega_j s_{ij}$, which is in the heart of the physics of turbulence. These quantities are among typical representing genuinely nonlinear processes in turbulence making it not amenable to quasi-nonlinear approaches such as RDT.

References

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