REYNOLDS NUMBERS NEAR THE ULTIMATE STATE OF TURBULENT RAYLEIGH-BÉNARD CONVECTION

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Abstract We report on measurements of the mean-flow Reynolds number Re_U and the rms fluctuation Reynolds number Re_V in turbulent Rayleigh-Bénard convection as a function of the Rayleigh number Ra for $4 \times 10^{11} \lesssim Ra \lesssim 2 \times 10^{14}$ and $Pr \simeq 0.8$. Both can be described by the same power law with an effective exponent $\zeta = 0.44$, in agreement with predictions for Re_U but in disagreement with predictions for Re_V.

We report results of Reynolds-number measurements, based on multi-point temperature correlation-function measurements and the elliptic approximation of He and Zhang [5, 7], for turbulent Rayleigh-Bénard convection (RBC) over the Rayleigh-number range $4 \times 10^{11} \lesssim Ra \lesssim 2 \times 10^{14}$ and for a Prandtl number $\mathrm{Pr} \simeq 0.8$. The sample was a right-circular cylinder with the diameter D and the height L both equal to 112 cm. The Reynolds numbers Re_U and Re_V were obtained from the mean-flow velocity U and the root-mean-square fluctuation velocity V respectively. Both were measured approximately at the mid-height of the sample and near (but not too near) the side wall close to a maximum of Re_U. The main contribution to Re_U came from a large-scale circulation in the form of a single convection roll with the preferred azimuthal orientation of its down flow nearly coinciding with the location of the measurement probes.

First we measured time sequences of $Re_U(t)$ and $Re_V(t)$ from short (10 s) segments which moved along much longer sequences of many hours. The corresponding probability distributions of $Re_U(t)$ and $Re_V(t)$ had single peaks and thus did not reveal significant flow reversals.

The two averaged Reynolds numbers determined from the entire data sequences were of comparable size and are shown in Fig. 1. For $2 \times 10^{12} \lesssim Ra < Ra_1^* \simeq 2 \times 10^{13}$ both Re_U and Re_V could be described by a power-law dependence on Ra with an exponent ζ close to 0.44. This exponent is consistent with several other measurements for the classical RBC state at smaller Ra and larger Pr and with the Grossmann-Lohse (GL) prediction for Re_U [2] (dashed line in Fig. 1) , but disagrees with the GL prediction $\zeta \simeq 0.33$ for Re_V [3] (solid line in Fig. 1). For $Ra \lesssim 2 \times 10^{12}$ the data for Re_U fell below the power-law fit at larger Ra; the reason for this is not clear.

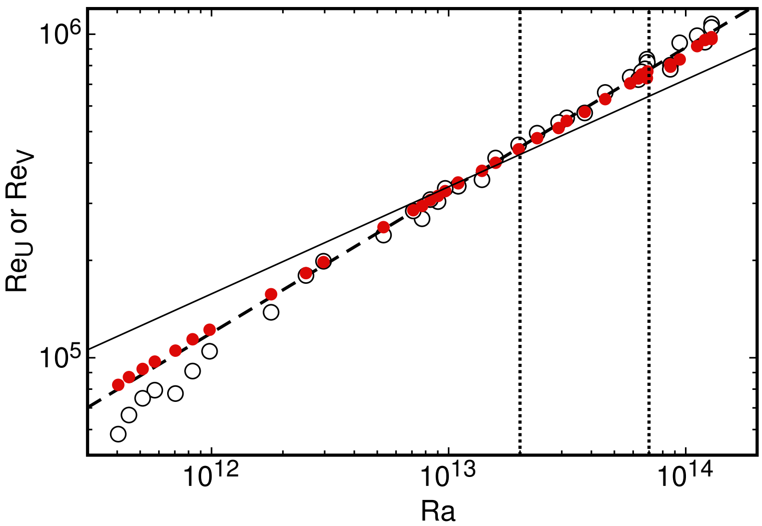

Figure 1. Re_U (open circles) and Re_V (solid red circles) as a function of Ra on logarithmic scales. The dashed line is the GL prediction for Re_U, with the pre-factor adjusted to fit the data. The solid line is the GL prediction for Re_V, with the pre-factor adjusted to fit the data near $Ra = 10^{13}$. The vertical dotted lines indicate our estimates of the locations of $Ra_1^* \simeq 2 \times 10^{13}$ and $Ra_2^* \simeq 7 \times 10^{13}$ (see Fig. 2 below).

In the bottom of Fig. 2 we show the reduced fluctuation Reynolds number $(Re_V / Pr^{\alpha_{GL}})/Re^{1/2}$ as a function of Ra. Here $\alpha_{GL} = -0.67$ is the exponent for the Pr dependence of Re_U predicted by GL. The term $Pr^{\alpha_{GL}}$ changes only very little with Ra since Pr is nearly constant. On this high-resolution graph one sees that at $Ra = Ra_2^* \simeq 7 \times 10^{13}$ the dependence of Re_V on Ra changed. For larger Ra $Re_V \sim Ra^{0.50 \pm 0.02}$, consistent with the prediction $\zeta = 1/2$ for Re_U

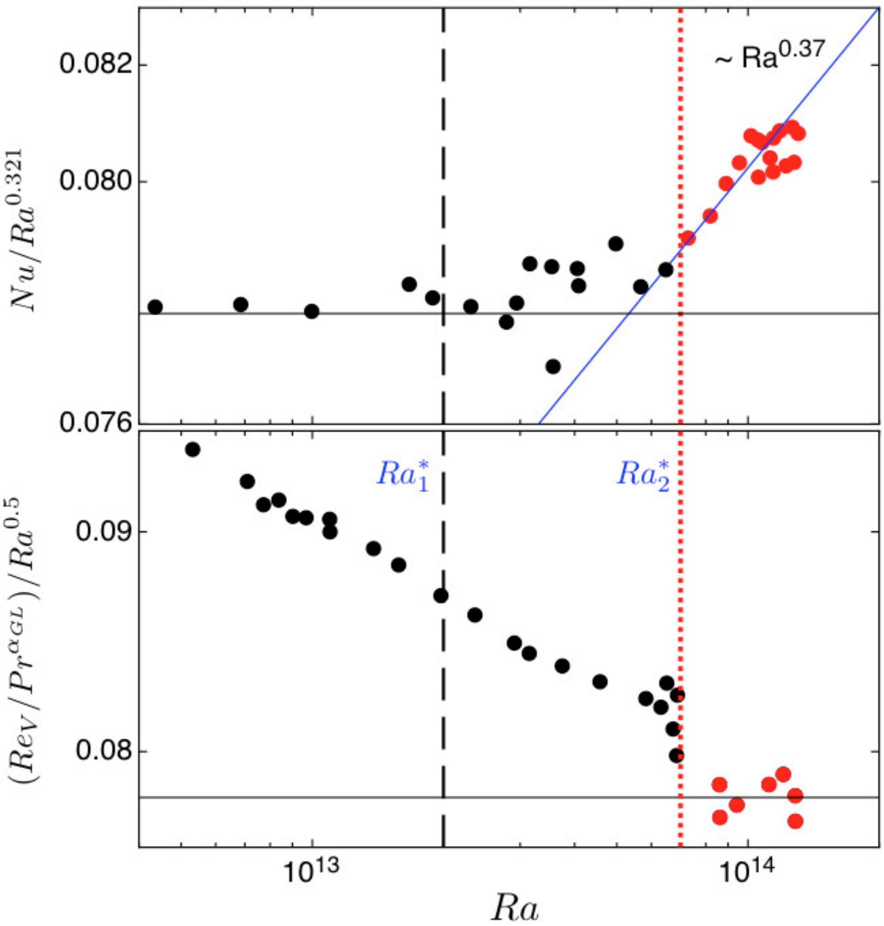

Figure 2. Top: The reduced Nusselt number $Nu/Ra^{0.321}$ and bottom: The reduced fluctuation Reynolds number $(Re_V/Pr^{\alpha_{GL}})/Re^{1/2}$, both as a function of Ra on a logarithmic scale. The vertical dashed and dotted lines represent our best estimate of the location of Ra_1^* and Ra_2^* respectively.

[4] in the ultimate state of RBC.

In the top of Fig. 2 we show recent measurements of Nu, also in a reduced form $Nu/Ra^{0.321}$, as a function of Ra. Also these measurements indicate that $Ra_2^* \simeq 7 \times 10^{13}$. We note that this value is much lower than the result $Ra_2^* \simeq 5 \times 10^{14}$ found for a sample with $\Gamma = 0.50$ [6, 1].

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