DRAG REDUCTION IN HOMOGENEOUS SHEAR FLOW TURBULENCE DILUTED WITH CONTRAVARIANT AND COVARIANT POLYMERS

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Abstract
This study conducts a multi-scale analysis on the drag reduction (DR) in the fluid flow diluted with the polymers. A mesoscopic description of ensemble of elastic dumbbells using Brownian dynamics (BDS) is connected to the macroscopic description for the solvent Newtonian fluid using DNS [1]. In Horiuti *et al.* [2], non-affinity in which the motion of dumbbells does not precisely correspond to macroscopically-imposed deformation was introduced and its effect on DR was elucidated in homogeneous isotropic turbulence (HIT). This work aims to carry out assessment on the influence of shear on DR in non-affine viscoelastic turbulence placed under constant mean shear. It is shown that the occurrence of DR and its mechanism are in agreement with those in HIT. More drastic DR is achieved when non-affinity is maximum than in the complete affine case. This difference is attributable to the convective motions of dumbbells. In the complete affine case, the connector vector of dumbbell is convected as a contravariant vector representing material line element, whereas, when non-affinity is the largest, it is convected as a covariant vector representing material surface element. In the latter case, the dumbbells directs outward perpendicularly on the planar structures and exert an extra tension on vortex sheet, which leads to attenuation of energy cascade, causing larger DR. The effect of presence of the streaks on the alignment of the dumbbells is discussed.

GOVERNING EQUATIONS FOR MOTION OF THE DUMBBELLS AND BDS-DNS RESULTS

Reduction of drag (DR) in the turbulent flow is critically important for engineering application. Addition of long-chain polymers into the Newtonian fluid flow is one of vital tools to accomplish DR. De Gennes [3] considered that, in the coil-stretch transition, stretched polymer chains behave elastically and it leads to modifications of turbulent cascade. In fact, occurrence of this DR is shown by solving the Navier-Stokes equation coupled with the viscoelastic polymer models to account for the effect of adding polymers [4]. It is generally assumed that the Newtonian fluid which surrounds the bead-spring configuration of the polymers moves affinely with an equivalent continuum [5]. In the fluid diluted with stretched polymer, however, molecular motions may not precisely correspond to the macroscopic deformation [3]. In Horiuti *et al.* [4], the Johnson-Segalman (JS) constitutive equation [5] was used to introduce non-affinity into the polymer stress. Remarkable enhancement of DR was achieved in forced homogeneous isotropic turbulence and pipe flow when non-affinity is maximum (slip parameter, $\alpha = 1.0$), compared with when complete affinity is assumed ($\alpha = 0.0$).

Limitation of DNS using the constitutive equation is in its inability to identify exact orientation of the dumbbells. To remedy this drawback, BDS-DNS approach [1] was developed by connecting a macroscopic description for the Newtonian turbulent flow whose evolution is pursued using DNS to the mesoscopic description of an ensemble of elastic dumbbells which are advected using the Brownian dynamics simulation (BDS). The polymer stresses incurred by the dumbbells are fed back into the Navier-Stokes equation. In Horiuti *et al.* [2], this method was modified so that the dumbbells are allowed to be advected non-affinely with the macroscopically-imposed deformation. It was applied to homogeneous isotropic turbulence. More drastic drag reduction was achieved when $\alpha = 1.0$ than $\alpha = 0.0$. In DNS of pipe flow using the JS equation, DR surpassing the Virk's maximum limit, which is similar to DR in addition of cationic surfactant, was achieved when $\alpha = 1.0$. It can be inferred that presence of mean shear may induce significant impact on DR.

This study aims to elucidate the effects of introduction of mean shear on DR using the modified BDS-DNS method. Stationary constant mean shear is established by an imposition of volume forcing [6]. In the homogeneous streamwise (x) and spanwise (y) directions, the periodic boundary conditions are used. At the boundaries in the shear (z) direction $z = 0, d$, the free-slip condition is imposed. Pseudospectral technique with a $3/2$-rule de-aliasing is used. A linear mean profile $\langle \overline{u}_x \rangle (z)/(Sd) = (z/d - 1/2)$ is approximated by a Fourier cosine series, and the force f_i is chosen such that the first 6 modes in the series remain constant in time. S denotes the constant mean shear rate. We carried out BDS-DNS using 128 grid points in the $x-$ and $y-$ directions, 33 in the $z-$ direction. The Reynolds number Sd^2/ν is set equal to 400, the Weisenberg number based on S is $Wi = 5.0$ and Wi based on the Kolmogorov-time scale is 10.76. Total number of dumbbells N_t is set equal to 10^7. The polymer stress tensor τ_{ij} due to the force acting on the fluid from the dumbbell are added to the Navier-Stokes equation. Since N_t is not sufficiently large, we adopted the replica method [1].

We denote the position vectors of each bead of the $n-$th dumbbell by $\boldsymbol{x}_1{}^{(n)}$ and $\boldsymbol{x}_2{}^{(n)}$ $(n = 1, 2, \cdots N_t)$. The governing equation for motion of the end-to-end connector vector $\boldsymbol{R}^{(n)} = \boldsymbol{x}_1{}^{(n)} - \boldsymbol{x}_2{}^{(n)}$ is given for the complete affine case as [1]

$$\frac{dR_i{}^{(n)}}{dt} = \{u_i(x_1{}^{(n)}) - u_i(x_2{}^{(n)})\} - \frac{1}{2\tau_s} \frac{R_i{}^{(n)}}{1 - (R_k{}^{(n)}/L_{max})^2} + \frac{r_{eq}}{\sqrt{2\tau_s}} \left((W_1{}^{(n)})_i - (W_2{}^{(n)})_i \right), \qquad (1)$$

where $u_i(x)$ denotes the velocity field of the solvent fluid, and the finitely extensible nonlinear elastic (FENE) model is applied to the elastic force. $(W_{1,2}{}^{(n)})_i$ is a random Gaussian force representing the Brownian motion of particles. $L_{max} (= 0.04)$ is the maximum length which the dumbbell can extend. $\tau_s (= 5.0)$ is the relaxation time. The equilibrium

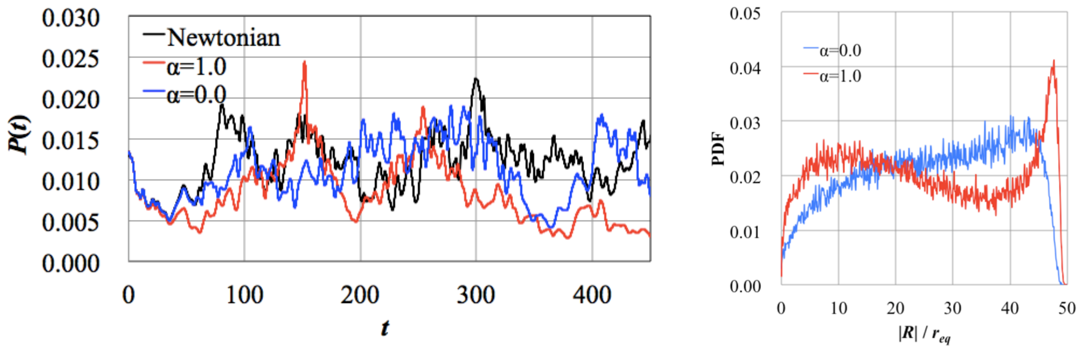

Figure 1. Left: Temporal variations in the work by the force $P(t)$; Right: Distribution of p.d.f of the length of the dumbbells $|\boldsymbol{R}|$.

length of the dumbbell r_{eq} is $L_{max}/50$. Equation (1) is solved with the equation for the center-of-mass vector $\boldsymbol{R}_g$ [1]. We introduce the non-affinity by allowing a slippage in the motion of polymer strand [4]. The velocity imposed at bead i, is given as $\boldsymbol{u}_i = \boldsymbol{u}_g + (\nabla \boldsymbol{u}_g) \cdot (\boldsymbol{R}_i - \boldsymbol{R}_g) - 2\alpha\{\boldsymbol{S}_t \cdot (\boldsymbol{R}_i - \boldsymbol{R}_g)\}$, where $\boldsymbol{u}_g$ denotes the velocity at the center, $\nabla \boldsymbol{u}_g$ is the velocity gradient tensor and $\boldsymbol{S}_t$ is the strain rate tensor. When $\alpha = 0.0$, Eq. (1) is analogous to the equation for evolution of a contravariant vector associated with a material line element of the fluid. When $\alpha = 1.0$, Eq. (1) becomes analogous to the equation for material surface element with its vector area $\boldsymbol{R}$.

Figure 1 (a) shows the temporal variations in the work due to the forcing $P(t)(\equiv \langle u_i' f_i \rangle)$ averaged in the whole computational box. Smaller $P(t)$ implies larger DR. When $\alpha = 0.0$ DR occurs in comparison to the Newtonian case, but DR in $\alpha = 1.0$ is more remarkable. Figure 1 (b) shows the distributions of the probability density function for the length of the dumbbells $|\boldsymbol{R}|$. The distribution from the $\alpha = 1.0$ case exhibits large concentration in the vicinity of L_{max}, i.e., stretching of the dumbbell in the case $\alpha = 1.0$ is larger than in $\alpha = 0.0$. Larger DR in $\alpha = 1.0$ is attributable to larger extension of the dumbbells because larger elastic energy is stored in the dumbbell than in $\alpha = 0.0$, and larger elastic effect ensues.

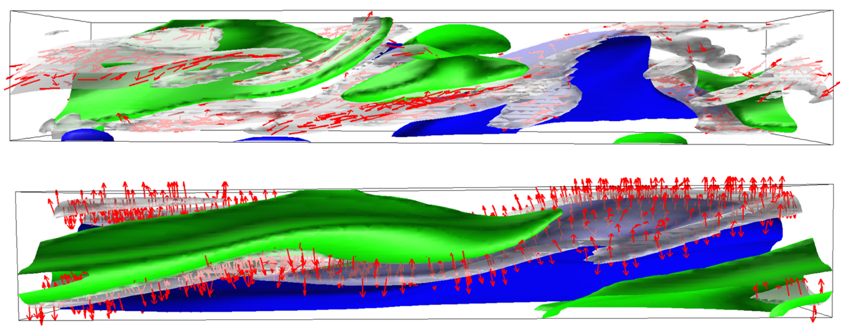

Figure 2. Three dimensional rendering of the isosurfaces of the vortex sheets (grey color), the streaks ($u' > 0$ in green, $u' < 0$ in blue) and the dumbbells (red arrows); Top: $\alpha = 0.0$, Bottom: $\alpha = 1.0$.

Figure 2 shows the configurations of vortex sheets, streaks and dumbbells. The sheets and high- and low-speed streaks are inclined against the $x-$ and $z-$ directions. Vortex sheet is placed between the streaks. When $\alpha = 0.0$, the dumbbells align selectively along the transverse direction of the sheet, and the principal polymer force opposes to the vortex stretching and growth of the vorticity is annihilated. When $\alpha = 1.0$, the dumbbells align preferentially in the direction normal to the surface of vortex sheets. The dominant principal polymer force directs outward perpendicularly on the vortex sheets. Pulling force is exerted on the vortex sheet and the sheet is under tension. Stretching and thinning of the sheet is reduced, leading to larger DR. These results agree with those for pipe flow obtained using the JS model [4].

References

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