ANGULAR STATISTICS OF LAGRANGIAN TRAJECTORIES

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Abstract The angle between subsequent particle displacement increments is evaluated as a function of the timelag in isotropic turbulence. It is shown that the evolution of the average of this angle contains two well-defined power-laws, reflecting the multi-scale dynamics of high-Reynolds number turbulence.

INTRODUCTION

In the Lagrangian reference frame, the spatio-temporal complexity of turbulence manifests itself through the spiraling chaotic motion of fluid particles, changing direction at every timescale. This directional change of Lagrangian tracers, as a function of the timelag between two observations, is the subject of the present work. Instantaneous measures of the curvature in turbulence have been investigated in the past five years for academic turbulent flows, both in three [2, 8] and in two space dimensions [6, 4]. The coarse grained curvature over a time interval was only recently introduced by Burov *et al.* [3]. More precisely, in this last work the directional change of a particle was introduced, and the characteristics of this new measure were investigated in various types of random walks. In the present work, we will show how this measure can characterize the time-correlation of the direction of a fluid particle in a turbulent flow. In particular will we show how the multi-scale character of a turbulent flow can be revealed by considering the timelag dependence of the directional change.

TIME-EVOLUTION OF THE COARSE-GRAINED ANGLE

We define the Lagrangian spatial increment as

$$\delta \boldsymbol{X}(\boldsymbol{x}_0, t, \tau) = \boldsymbol{X}(\boldsymbol{x}_0, t) - \boldsymbol{X}(\boldsymbol{x}_0, t-\tau) \tag{1}$$

where $\boldsymbol{X}(\boldsymbol{x}_0, t)$ is the position of a fluid particle at time t, passing through point $\boldsymbol{x}_0$ at the reference time $t = t_0$ and advected by a velocity field $\mathbf{u}$, i.e. $d\boldsymbol{X}/dt = \mathbf{u}$. The cosine of the angle $\Theta(t, \tau)$ between subsequent particle increments, introduced in [3], is

$$\cos(\Theta(t,\tau)) = \frac{\delta \boldsymbol{X}(\boldsymbol{x}_0, t, \tau) \cdot \delta \boldsymbol{X}(\boldsymbol{x}_0, t+\tau, \tau)}{|\delta \boldsymbol{X}(\boldsymbol{x}_0, t, \tau)|\, |\delta \boldsymbol{X}(\boldsymbol{x}_0, t+\tau, \tau)|}. \tag{2}$$

The angle is illustrated in Figure 1 (left). Rather than considering its instantaneous evolution, its averaged absolute value is of particular interest in an isotropic random velocity field. The ensemble average will be denoted in the following by

$$\theta(\tau) \equiv \langle |\Theta(t,\tau)| \rangle\,. \tag{3}$$

For short time lags, $\theta(\tau)$ should be close to zero, whereas for times long compared to the correlation time associated with the spiraling motion $\theta(\tau)$ should tend to $\pi/2$ by symmetry.
For short times the instantaneous angle $\Theta(\tau, t)$ is related to the curvature κ (see Figure 1 (left)) by the relation

$$\kappa(t) = \lim_{\tau \to 0} \frac{|\Theta(t,\tau)|}{2\tau \|\boldsymbol{u}(t)\|}, \tag{4}$$

with $\boldsymbol{u}$ being the velocity. How the angle varies in between the short and long-time limits is the main subject of the present work and we will show that the dependence of $\theta(\tau)$ on the timelag contains the signature of the multi-scale dynamics of a turbulent flow.
The database used to investigate the behaviour of $\theta(\tau)$ is described in [5, 7]. The simulation was carried out using standard pseudo-spectral techniques, following 8.10^6 fluid particles in a statistically stationary isotropic turbulent flow during 5.8 integral timescales in a periodic cube of dimension 2π. The resolution is 1024^3 gridpoints. The integral timescale is 2.1 and the Kolmogorov timescale $\tau_K = (\nu/\epsilon)^{1/2} = 0.036$, where $\epsilon = 0.31$ is the mean dissipation rate and $\nu = 4.10^{-4}$ the kinematic viscosity. The Lagrangian integral timescale is of the order of the Eulerian integral timescale. The Taylor-scale Reynolds number is $R_\lambda = 225$.
Figure 1 (right) shows $\theta(\tau)$ in double-logarithmic representation. The angle increases monotonuously from zero to $\pi/2$, and this latter value is approached for values of τ of the order of the Lagrangian integral timescale. Two power-laws can

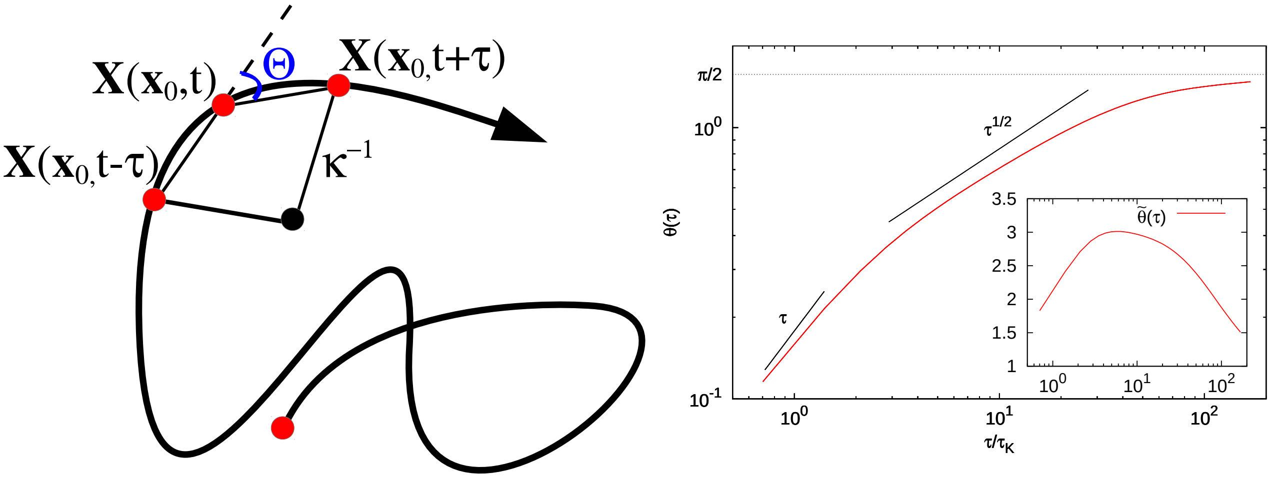

Figure 1. Left: definition of the angle between subsequent Lagrangian particle increments. Right: the average angle θ as a function of the timelag τ normalized by the Kolmogorov time-scale τ_K. In the inset the compensated angle, $\tilde{\theta}(\tau) \equiv \theta(\tau)\sigma_u/(\epsilon\tau)^{1/2}$ is plotted..

be identified in this graph, with a cross-over around twice the Kolmogorov timescale. The origin of these power-laws is elucidated in the full paper [1].

It is derived that, within the framework of Kolmogorov's 1941 inertial range theory, the scaling should be given for τ small with respect to the smallest Lagrangian time-scale, the Kolmogorov scale, by

$$\theta(\tau) \approx 2\tau\frac{\sigma_a}{\sigma_u} \quad \text{for } \tau \ll \tau_K. \tag{5}$$

where σ_a and σ_u are the total rms perpendicular acceleration and velocity, respectively. The linear relation between $\theta(\tau)$ and τ is well observed in Figure 1 (right).

At timescales larger than τ_K, but smaller than T, i.e., in the inertial interval, we obtain, assuming dimensional scaling for the perpendicular acceleration, that

$$\theta(\tau) \sim \tau^{1/2}\frac{\epsilon^{1/2}}{\sigma_u} \sim \left(\frac{\tau}{T}\right)^{1/2} \quad \text{for } \tau_K \ll \tau \ll T. \tag{6}$$

Again, this scaling is observable in Figure 1 (right), even though the power-law is less well present than in the dissipation range. This is better appreciated by considering the compensated angle, $\tilde{\theta}(\tau) \equiv \theta(\tau)\sigma_u/(\epsilon\tau)^{1/2}$, plotted in the inset of the figure.

In the full paper [1] it is shown that the probability density function of the directional change is close to self-similar and well approximated by an analytically derived model assuming Gaussianity and independence of the velocity and the Lagrangian acceleration.

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