TURBULNENT STRUCTURES IN AN OPTIMAL TAYLOR-COUETTE FLOW BETWEEN TWO COUNTER-ROTATING CYLINDERS

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Abstract Taylor-Couette flow with two independently counter-rotating cylinders is investigated. Direct numerical simulation is applied to study flow structure and angular velocity transport for $\eta = 0.714$ at optimum and fully turbulent regime. The main purpose is to study the coherent structure in both axial and radial directions and its contribution to angular velocity transport in optimum condition. Visualizing the vortical structure (and other structural parameters) with the distribution of azimuthal velocity and ω-Nusselt number leads to a better understanding of turbulent flow structure at optimum condition comparing to non-optimum condition.

COMPUTATIONAL DETAILS AND INITIAL RESULTS

In the present work, Taylor-Couette (TC) flow is simulated between two counter-rotating cylinders. In this system the fluid flows between two independently rotating cylinders which are coaxial (figure 1a). TC flow has many similarities to Rayleigh-Benard flow which is a thermal flow in a fluid between two plates (a hot plate at the bottom and a cold plate above the fluid) [1]. Both systems have been used to study many concepts in fluid dynamics like instability, pattern formation and turbulence [2]. Due to the similarities to Rayleigh-Benard flow, similar relationships have been applied to study TC flow. The driving force of the system is defined as the Taylor number (Ta)

$$Ta = \frac{1}{4}\frac{\sigma(r_o - r_i)^2(r_o + r_i)^2(\omega_i - \omega_o)^2}{\nu^2}$$

or shear Reynolds number $Re_s = 2 \times \left|\eta Re_o - Re_i\right|/(1+\eta)$ and global response of the system is $Nu_\omega = J^\omega / J_0^\omega$. $J^\omega = r^3(<u_r\omega>_{A,t} - \upsilon\partial_r <\omega>_{A,t})$ is the angular velocity transport in the system and J_0^ω is the angular velocity transport for laminar and purely azimuthal flow. In the TC flow system the torque applied on cylinders is equal to the viscose dissipation in the fluid [3] and the relationship between the global response of the system with the energy dissipation is $Nu_\omega = T/(2\pi\rho J_0^\omega)$, *T: torque*. The condition for maximum energy dissipation in the system is known as the optimum condition. Therefore, in the optimal TC flow, we have maximum Nu_ω and J^ω which corresponds to maximum angular velocity transport.

Many studies have been made to figure out the system statistical conditions and specifications in the optimum condition [2,4,5]. It has been shown that achieving the optimum condition strongly depends on the Taylor number, the radius ratio $\eta = r_i / r_o$ and the Rossby number $Ro = (\left|\omega_i - \omega_o\right| r_i)/(2\omega_o d)$ [4]. However; increasing Ta, Ro_{opt} saturates to a specific value for every radius ratio. Increasing Ta (consequently Re_s) also leads to the transition to fully turbulent (ultimate) regime where both boundary layers adjacent to the cylinders and bulk of the fluid are turbulent [6,7]. Prior to the ultimate regime, the angular velocity transport is mainly supported by large scale Taylor vortices in the bulk of the fluid which are confined by a mixture of both laminar and turbulent boundary layers [6]. However; Taylor rolls in the bulk cause an axial pressure gradient too which in turn affects the boundary layers and the rolls. Increasing the driving force sufficiently, axial pressure gradient and consequently Taylor rolls vanish and both the boundary layers and the bulk become turbulent [8]. Reaching the fully turbulent regime depends on the radius ratio and boundary conditions (angular velocities of the cylinders). Depending on the radius ratio and the Rossby number, remnants of Taylor rolls are still visible in the ultimate regime [8]. Thus more investigation is needed to study flow structures especially in fully turbulent regime.

In this work, we are going to study turbulent structures between two counter-rotating cylinders with the radius ratio $\eta = 0.714$ at the optimum condition and fully turbulent regime using the dataset obtained by direct numerical simulation. The flow is simulated by solving the Navier-Stokes equation in a rotating frame. The rotating frame rotates with the angular velocity ω_o so the boundary conditions are $u_{\theta,o} = 0$ and $u_{\theta,i} = r_i(\omega_i - \omega_o)$. The fully implicit decoupling method is applied for solving the governing equations [9] and periodic boundary conditions are applied in the x and θ directions so the end effects present in experiments are not present in the simulation.

Turbulent structures in both radial and axial directions are studied to investigate its contribution to the angular velocity transport at the optimum condition. Studying the attached/detached eddies and vortical structures between cylinders

would be helpful to understand and visualize the remnants of Taylor rolls in the bulk and the interaction of bulk and boundary layers. The turbulent boundary layers structures will also be analyzed in the fully turbulent optimum condition. Along with the distribution of Nu_ω and azimuthal velocity some statistical results will also be studied to understand the flow structure better (like quadrant analysis and log layer structure in boundary layers and two point correlation of velocities for different points in both radial and axial condition).
Figure 1b shows the instantaneous velocity field with contours of the retro/prograde vortices for the 1/6th of the same geometry at lower Taylor number (T = 2.39x10^7) and R_{opt} = -0.12 (flooded contours show 30% of the maximum and minimum λ_{ci}) [4,10]. As seen, there are no vortices near the inner cylinder while there is a cluster of the prograde vortices near the outer cylinder after a very small distance which is vacant of vortices. The results are reasonable at lower *Ta* where the flow is not fully turbulent, a large area in the gap is occupied by Taylor rolls (and no vortices) [8]. However; as the outer cylinder is rotating in the reverse direction there is a Rayleigh stable area near the outer cylinder [7] which is filled with the pograde vortices. The small vacant area near the outer cylinder is the boundary layer which is not fully turbulent yet. The edge of area filled with the prograde vortices falls to the neutral surface (zero azimuthal velocity). As the prograde spanwise vortices rotate in the same sense as the main shear[10], it can be conjectured that there is a strong shear between the neutral surface and the boundary layer near the outer cylinder due to the rotation of the outer cylinder in the reverse direction.

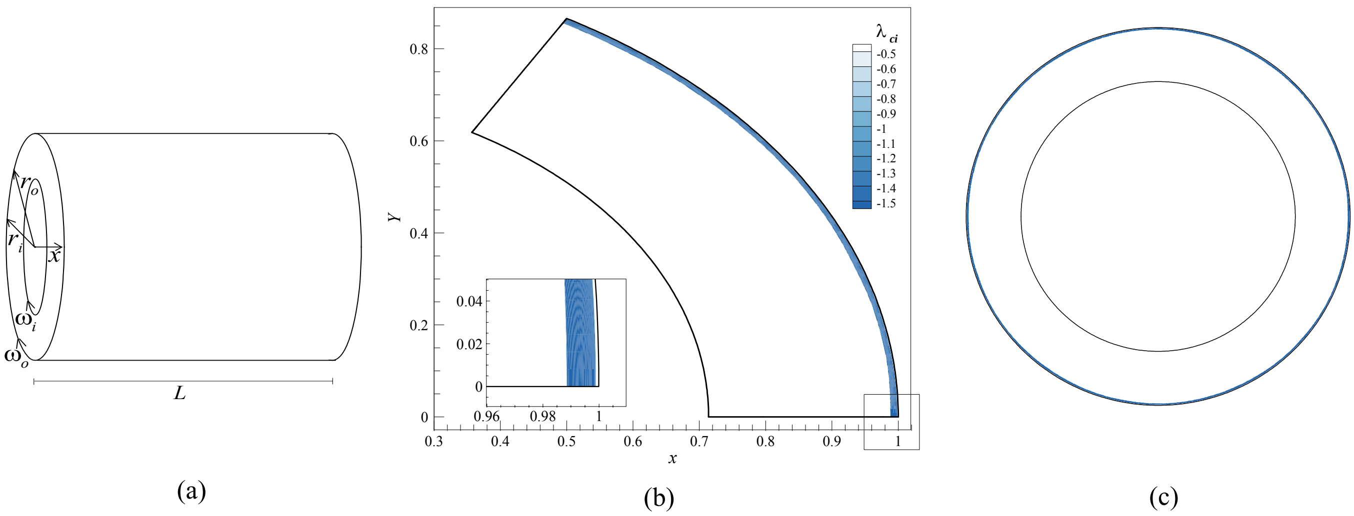

Figure 1. (a)Schematic diagram of the computational domain, (b)flooded contours of the spanwise swirling strength (30% of the maximum and minimum λ_{ci}) in the Galilean frame, the inset shows the flow structure and the boundary layer near the outer cylinder, (c) the area occupied by prograde vortices in the whole domain

References

[1] B. Eckhardt, S. Grossmann, and D. Lohse. Torque scaling in turbulent taylor-couette flow between independently rotating cylinders. *J. Fluid Mech*., **581**:221–250, 2007.
[2] R. Ostilla-Monico, R. J. A. M. Stevens, S. Grossmann, R. Verzicco and D. Lohse. Optimal Taylor-Couette flow: direct numerical simulation. *J. Fluid Mech*., **719**:14–46, 2013.
[3] D. P. Lathrope, J. Fineberg and H. L. Swinney. Turbulent flow between concentric rotating cylinders at large Reynolds number, *Phys. Rev. Lett.* **68:** 1515-1518, 1992.
[4] R. Ostilla-Monico, S. G. Huisman, T. J. G.. Jannink, D. P. M. Van Gil, S. Grossmann, C. Sun and D. Lohse. Optimal Taylor-Couette flow: radius ratio dependence. *J. Fluid Mech*., **747**:1–29, 2014.
[5] D. P. M. Van Gils, S. G., Huisman, S. Grossmann, C. Sun and D. Lohse. Optimal Taylor–Couette turbulence. *J. Fluid Mech.*, **706 :** 118–149,2012.
[6] R. Ostilla-Monico, E. P. Van Der Poel, R. Verzicco, S. Grossmann and D. Lohse. Boundary layer dynamics at the transition between the classical and ultimate regime of Taylor-Couette flow. *Phys. of Fluids.,* **26:**015114, 2014.
[7] H. J. Brauckmann, and B. Eckhardt. Intermittent boundary layers and torque maxima in Taylor–Couette flow. *Phys. Rev. E*., **87:** 033004, 2013.
[8] R. Ostilla Monico, E. P. Van Der Poel, R. Verzicco, S. Grossmann and D. Lohse. Exploring the control parameter space for the transition to the ultimate regime of Taylot-Couette flow. *J. Fluid Mech.*, in press.
[9] K..Kim, S. J. Baek and H. J. Sung. An implicit velocity decoupling procedure for the incompressible Navier–Stokes equations. *Intl J. Numer. Meth. Fluids* **38:** 125–138, 2002.
[10] V. K. Natrajan, Y. Wu and K. T. Christensen. Spatial signatures of retrograde spanwise vortices in wall turbulence. *J. Fluid Mech.*, **574 :** 155–167,2007.