DIRECTION CHANGE OF FLUID PARTICLES IN CONFINED TWO-DIMENSIONAL TURBULENCE

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Abstract The directional change of a fluid particle can be measured by the angle between two subsequent particle displacement increments. At small values of the time-increment the so-defined angle is proportional to the curvature of the trajectory. At large values this coarse-grained curvature should be affected by the presence of solid no-slip walls around the flow domain. We compare homogeneous and confined two-dimensional turbulent flows and show that the PDF of the angle is indeed strongly modified by the presence of walls.

INTRODUCTION

The Lagrangian point of view is in many aspects the most natural way to obtain understanding of turbulent transport and mixing. For instance, the characterization of particle trajectories is of importance to describe and quantify the topology of fluid motion. In previous investigations the curvature was used to characterize the particle trajectories in turbulent flows, both in three [3, 9] and in two space dimensions [8, 6]. In a recent investigation [2] we applied a measure introduced by Burov et al. [4] to three-dimensional turbulent flows. This measure is the directional change of a particle, by considering the angle between subsequent particle displacement increments. Intuitively one can suspect that at long-times the curvature should be affected by the presence of solid boundaries, and we investigate this comparing the timelag dependence of the directional change in periodic and confined turbulent flows.

NUMERICAL METHOD

We consider two distinct geometries in this study: a square domain with double periodic boundary conditions and a circular domain with no-slip boundary conditions. Incompressible turbulent flow is governed by the two-dimensional Navier-Stokes equations written in dimensionless form

$$\frac{\partial \omega}{\partial t} + \boldsymbol{u} \cdot \nabla \omega = \nu \nabla^2 \omega + F_\omega, \tag{1}$$

where, $\boldsymbol{u}$ is the velocity, $\omega = \nabla \times \mathbf{u}$ is the vorticity and ν is the kinematic viscosity. The turbulence is kept statistically stationary by a random isotropic stirring in both flows, and a large scale friction term in the periodic case. Those effects are represented by F_ω. In the wall-bounded case no friction-term is needed to attain a stationary flow since the walls act as a source of entrophy, allowing the flow to sufficiently dissipate the kinetic energy at large scales. The boundary conditions are enforced using the volume penalization method [1, 7] implemented in a classical pseudo-spectral method, fully dealiased. The particle trajectories are calculated by interpolating the Eulerian quantities and by using a second order Runge-Kutta scheme for time integration. The Lagrangian statistics are computed by ensemble averaging over 10^4 trajectories. The Reynolds number based on the Taylor microscale is for both geometries of order $R_\lambda = 700$. More details can be found in [5]. The confined flow we consider is shown in Figure 1 (left).

ANGULAR STATISTICS

We define the Lagrangian spatial increment as

$$\delta \mathbf{X}(\mathbf{x}_0, t, \tau) = \mathbf{X}(\mathbf{x}_0, t) - \mathbf{X}(\mathbf{x}_0, t - \tau) \tag{2}$$

where $\mathbf{X}(\mathbf{x}_0, t)$ is the position of a fluid particle at time t, passing through point $\mathbf{x}_0$ at the reference time $t = t_0$ and advected by a velocity field $\mathbf{u}$, i.e., $d\mathbf{X}/dt = \mathbf{u}$. The cosine of the angle $\Theta(t, \tau)$ between subsequent particle increments, introduced in [4], is

$$\cos(\Theta(t, \tau)) = \frac{\delta \mathbf{X}(\mathbf{x}_0, t, \tau) \cdot \delta \mathbf{X}(\mathbf{x}_0, t + \tau, \tau)}{|\delta \mathbf{X}(\mathbf{x}_0, t, \tau)| \; |\delta \mathbf{X}(\mathbf{x}_0, t + \tau, \tau)|} \tag{3}$$

Figure 1 (right) shows the probability density functions of this angle. For small increments where the angle should on average be small, we observe for both flows a peaked distribution of the angle, centered around zero. For increments large compared to the Lagrangian correlation time of the flow, the angle should become completely randomly oriented and an

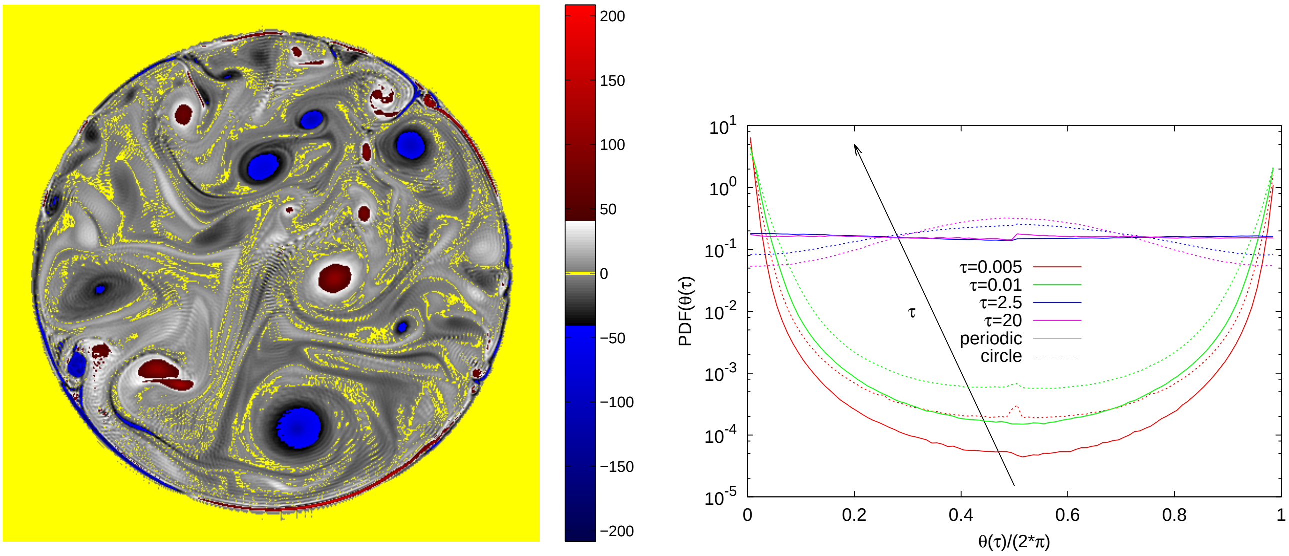

Figure 1. Left: visualization of the vorticity in the confined two dimensional turbulent flow. Right: PDF of the directional change compared between the two types of flow.

equi-distribution over all possible angles should be observed. This is indeed the case for the periodic flow. However, in the presence of walls the angle is importantly affected and a bump around $\theta = \pi$ is observed.

In order to understand this we have computed the angle between two connected line segments by randomly choosing three points in a circle. Computing the PDF of this angle over a large number of realizations perfectly reproduced the shape of the PDF in Figure 1(b) for large time-increments. This shows that the shape of the PDF for very large values of the time-increment can be explained by purely geometrical arguments, independent of the characteristics of the turbulent flow.

In the full talk we will further focus on the periodic case. We will try to analytically predict the shape of the PDFs for short time intervals using Taylor-expansions and scaling relations for the Lagrangian acceleration. We will further quantify the average of the absolute value of the angle as a function of the time-lag and investigate whether classical scaling arguments can correctly describe the phenomenology of the directional change in two-dimensional turbulence.

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