EXCESS STATISTICS WITH APPLICATIONS TO TURBULENCE IN MARINE ENVIRONMENTS

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Abstract Studies of excess statistics are standard in signal analysis, addressing basic questions like average frequencies of level crossings in a random signal, and average time durations between an up- and a down-crossing. The results can be applied to estimating the noise experienced by aquatic microorganisms in a turbulent environment. This problem has particular interest in relations to predators and prey that rely solely on signals transmitted through the surrounding turbulent flow. It is possible to give estimates for how often a predator can mistake turbulent induced noise for a signal from prey in given turbulence conditions. Such mistakes can be observed as unmotivated attacks. Similarly, prey can mistake noise for presence of a predator, giving rise to seemingly unmotivated observable escape responses.

INTRODUCTION

Small aquatic organisms, plankton, move at small velocities, on average, and induce moderate velocity perturbation in their environment. Such disturbances will be detected by other organisms (predators or prey) by their sensing organs, for instance the antennae shown in Figure 1. At these low Mach numbers the flows can be taken to be incompressible so that pressure disturbances can be ignored. The organisms discussed here are close to be neutrally buoyant, and inertia effects are small. Consequently, we expect that plankton senses mostly velocity differences rather than absolute velocities. As a characteristic quantity we introduce the velocity difference $\boldsymbol{v} = \boldsymbol{u}_0 - \boldsymbol{u}_b$ at maximum separation b, see Figure 1. We here consider only the longitudinal velocity difference component. The velocity disturbances induced by moving plankton will typically have orders of magnitude of a few $mm\ s^{-1}$ and observed durations of 0.1 – 1.0 *s*.

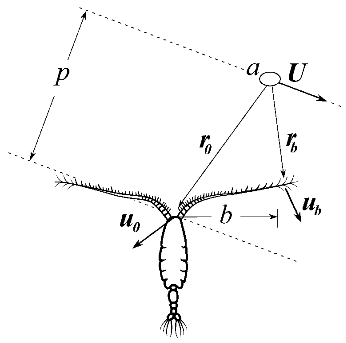

Figure 1. Diagram for the velocity perturbation near a predator due to relative prey motions (after [1]). Prey is indicated by its size *a*. It moves with a velocity *U* at an impact parameter *p* with respect to the predator, inducing velocity disturbances $\boldsymbol{u}_0$ and $\boldsymbol{u}_b$ at the two ends of the predator's antennae with length *b*. For many cases we find that $b<\eta_0$, where η_0 is an effective Kolmogorov length. The flow dynamics on this scale should be described by expressions relevant for the viscous subrange.

The turbulent environment is also giving rise to velocity differences along the antennae and this effect can, depending on parameters, appears as a noise that masks the signal from prey or predator. In the present study we will demonstrate means for estimating the character of the noise from a turbulent environment. The characteristics of this turbulence vary significantly in the oceans [2], as summarized by typical parameters listed in Table 1. We find that it will be mostly the viscous subrange that is relevant for the present problem.

Location	Specific energy dissipation rate, ε	Modified Kolmogorov length, η_0	Kolmogorov time, $\tau_K = (\nu/\varepsilon)^{1/2}$	Kolmogorov velocity, $u_K = (\nu\varepsilon)^{1/4}$
Open ocean	$10^{-4} – 1.0$ mm^{2}s^{-3}	130 – 13 mm	100 – 1.0 s	$10^{-1} – 1.0$ mm s^{-1}
Shelf	$10^{-1} – 1.0$ mm^{2}s^{-3}	26 – 13 mm	3.0 – 1.0 s	0.5 – 1.0 mm s^{-1}
Coastal zone	$10^{-1} – 10^2$ mm^{2}s^{-3}	26 – 2.6 mm	3.0 – 0.1 s	0.5 – 3.0 mm s^{-1}
Tidal front	10 mm^{2}s^{-3}	6.5 mm	0.3 s	1.7 mm s^{-1}

Table 1. Basic ocean data.

We assume that a recognizable signal from a moving predator or prey can be characterized by a minimum velocity disturbance U_* and by a characteristic time duration t_*. The two basic questions considered in the present study is 1) how often will the noise from the turbulent environment give perturbation exceeding U_*, and 2) what is the average time duration the signal $\upsilon(t)$ spends above U_* after such an upwards crossing? Both quantities can be estimated analytically. It is found that the basic information needed is found in the joint probability density $P(\upsilon,\upsilon')$ of the random process $\upsilon(t)$, where $\upsilon' \equiv d\upsilon/dt$. This joint probability density is here found by analyzing data from numerical simulations.

ANALYSIS

Considering the fluctuating velocity difference to be a random signal, it is argued [3] that the average number of times n of crossing some level U_* within a time interval dt can be found by dividing the expected amount of time spent in the interval $d\upsilon$ for a given υ' in the time dt by the time $\tau = d\upsilon/\upsilon'$ that is required to cross the interval $d\upsilon$ for the given υ'. This argument gives [3] the well-known result $n(U_*,\upsilon') = \upsilon' P(U_*,\upsilon') d\upsilon' dt$ for a given υ'. The average number of level crossings, irrespective of the magnitude of υ', is found by integrating over all positive υ' to give the average crossing frequency $\nu = \int_0^\infty \upsilon' P(U_*,\upsilon') d\upsilon'$. The average time duration spent from one up-crossing to the next down-crossing can be estimated [4] as $\langle \mathcal{T} \rangle = \int_{U_*}^\infty P(\upsilon) d\upsilon / \nu$. Particularly simple results can be obtained for Gaussian random processes, but we find that $\upsilon(t)$ is generally not such a process due to the intermittent features of the turbulence.

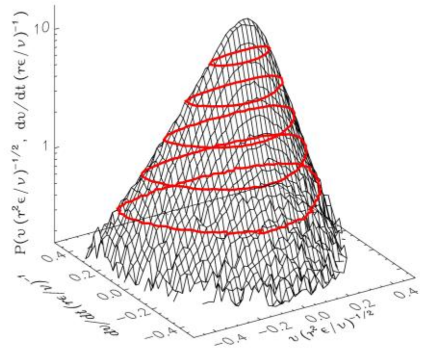

Figure 2. Joint probability density for normalized velocity differences and velocity difference derivatives, for separations b smaller than the Kolmogorov length.

The information needed to determine ν and $\langle \mathcal{T} \rangle$ is found in the joint probability density $P(\upsilon,\upsilon')$. There are no analytical results for this quantity, but it can be determined by analyzing numerical simulation data [5,6]. Our results are summarized (using normalized units) in Figures 2 and 3. We have particular interest in the viscous subrange, but analyzed also separations b (see Figure 1) in the inertial subrange. The results have relevance also for short fibers in turbulent environments.

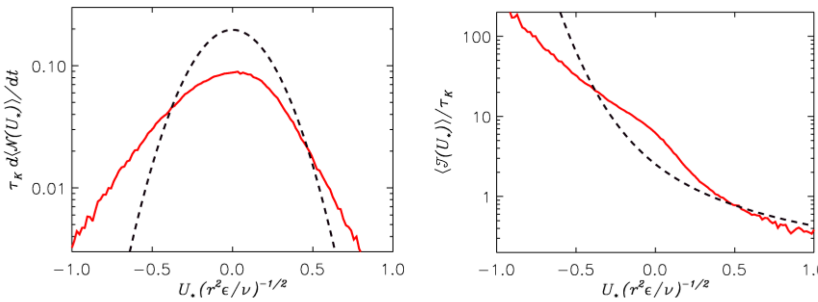

Figure 3. Normalized average level crossing frequency and normalized average excess time-duration derived from the joint probability density in Figure 2. Dashed lines indicate results from an equivalent Gaussian process.

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