# SIMILARITIES BETWEEN 2D AND 3D CONVECTION FOR LARGE PRANDTL NUMBER

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*Abstract* Using direct numerical simulations of Rayleigh-Bénard convection (RBC), we perform a comparative study of the spectra and fluxes of energy and entropy for large and infinite Prandtl numbers in two (2D) and three (3D) dimensions. We observe close similarities between the 2D and 3D RBC, in particular the kinetic energy spectrum $E_u(k) \sim k^{-13/3}$, and the entropy spectrum exhibits a dual branch with a dominant $k^{-2}$ spectrum. We showed that the dominant Fourier modes in the 2D and 3D flows are very close.

## INTRODUCTION

Thermal convection plays a crucial role in the heat transport in the interiors of stars and planets, as well as in many engineering applications. Earth's mantle and viscous fluids have large Prandtl number. Their convective flow is dominated by sharp "plumes". Schmalzl *et al.* [1, 2] and van der Poel *et al.* [3] showed that for large Prandtl number, the flow structures and global quantities, e.g., the Nusselt number and Reynolds number, exhibit similar behaviour for three dimensions (3D) and two dimensions (2D). Also, the energy and entropy spectra are important quantities in Rayleigh-Bénard convection, and have been studied extensively for various Prandtl numbers [4, 5, 6]. Pandey *et al.* [6], in their numerical simulations for very large Prandtl numbers in three dimensions, reported that the kinetic energy spectrum $E_u(k)$ scales as $k^{-13/3}$, and the entropy spectrum $E_\theta(k)$ shows a dual branch with a dominant $k^{-2}$ spectrum. We analyze the flow behavior of 2D and 3D flows for large Prandtl numbers, and show that the large-scale Fourier modes of 2D and 3D RBC are very similar. We also observe that the kinetic energy spectrum $E_u(k) \sim k^{-13/3}$, and the entropy spectrum exhibits a dual branch with a dominant $k^{-2}$ spectrum in 2D and 3D RBC for very large Prandtl numbers.

## SIMULATION METHOD AND RESULTS

We perform direct numerical simulations (DNS) for Prandlt numbers $10^2$, $10^3$, and $\infty$ and Rayleigh numbers in the range $10^5$ to $5 \times 10^8$. The aspect ratio of the box is $2\sqrt{2} : 1$. We employ stress free boundary condition for velocity field, and conducting boundary condition for the temperature field for the horizontal plates. However, for the vertical side walls, periodic boundary condition is used for both the temperature and velocity fields. The fourth order Runge-Kutta method is used for the time advancement, and 2/3 rule for dealiasing. We use pseudospectral code TARANG [7] for all our simulations.

In Fig. 1, we plot the normalized kinetic spectrum $E_u(k)k^{13/3}$ for ($\mathrm{Pr} = 100, \mathrm{Ra} = 10^7$), and ($\mathrm{Pr} = \infty, \mathrm{Ra} = 10^8$) both for 2D and 3D RBC. The figure illustrates that the energy spectrum for 2D and 3D are quite close. We also plot the entropy spectrum $E_\theta(k)$ for the aforementioned parameters for 2D and 3D RBC. Clearly they have very similar behaviour. Note that the entropy spectrum exhibits a dual spectrum, with the top curve ($E(k) \approx k^{-2}$) representing the $\hat{\theta}(0, 0, 2n)$ modes, whose values are close to $-1/(2n\pi)$ (see Mishra and Verma [5]). The lower curve in the spectrum corresponds to the modes other than $\hat{\theta}(0, 0, 2n)$, is somewhat flat.

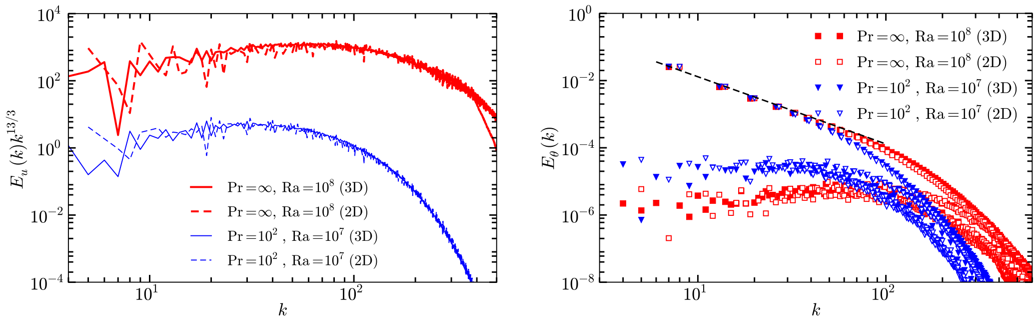


**Figure 1.** (left Panel) Compensated Kinetic energy spectrum $E_u(k)k^{13/3}$ vs $k$, (right panel) Entropy energy spectrum $E_\theta(k)$ vs $k$

The nonlinear interactions induce kinetic energy and entropy transfers from larger length scales to smaller length scales that results in kinetic energy and entropy fluxes. We compute the spectra and fluxes of energy and entropy for 2D and 3D RBC for large and infinite Prandlt numbers and compare them. We show that these quantities are very close to each other for 2D and 3D RBC.

**Table 1.** Comparison of the four most dominant entropy Fourier modes in 2D and 3D RBC for $\mathrm{Pr} = \infty$ and $\mathrm{Ra} = 10^7$.

| Mode (3D) $(n_x, n_y, n_z)$ | $E_\theta^{\mathrm{mode}}/E_\theta^{3D}$ (%) | $E_u^{\mathrm{mode}}/E_u^{3D}$ (%) | Mode (2D) $(n_x, n_z)$ | $E_\theta^{\mathrm{mode}}/E_\theta^{2D}$ (%) | $E_u^{\mathrm{mode}}/E_u^{2D}$ (%) |
|---|---|---|---|---|---|
| (1,0,1) | 0.018 | 19.6 | (1,1) | 0.020 | 20.4 |
| (3,0,1) | 0.011 | 2.05 | (3,1) | 0.018 | 3.39 |
| (1,0,3) | 0.011 | 0.046 | (1,3) | 0.011 | 0.046 |
| (3,0,3) | 0.003 | 0.039 | (3,3) | 0.007 | 0.095 |

The kinetic energy flux $\Pi_u$ for $\mathrm{Pr} = \infty$ is zero due to the absence of nonlinearity. However $\Pi_u$ is expected to be small (in normalized units of ours) for large Pr. In Fig. 2 we plot the kinetic energy flux $\Pi_u(k)$ for $\mathrm{Pr} = 10^2$ and $10^3$ for 2D and 3D RBC. As expected, the $\Pi_u$ are small for all four cases. Interestingly, the kinetic energy flux for 2D RBC is negative at small wavenumbers, which is reminiscent of 2D fluid turbulence [8]. The KE flux for 3D RBC is positive almost everywhere. Thus, the KE fluxes for 2D and 3D RBC are somewhat different, but they play insignificant role in the large- and infinite Prandlt number RBC. Hence, we can claim that a common gesture for the large Pr 2D and 3D RBC is that $\Pi_u \to 0$. We also compute the entropy flux for $(\mathrm{Pr} = 100, \mathrm{Ra} = 10^7)$, and $(\mathrm{Pr} = \infty, \mathrm{Ra} = 10^8)$ for both 2D and 3D RBC and show them in Fig. 2. Clearly, the behaviour of 2D and 3D RBC are very similar, with a constant entropy flux in the inertial range.

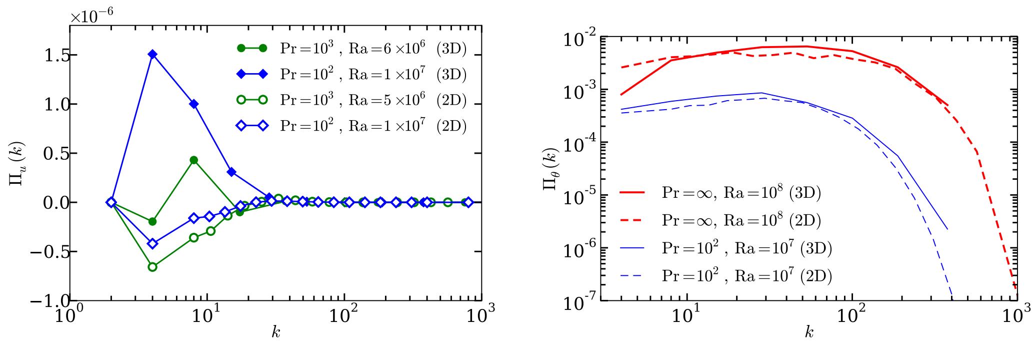


**Figure 2.** (left panel) Kinetic energy flux $\Pi_u$ vs $k$, (right panel) Entropy flux $\Pi_\theta$ vs $k$

In Table 1, we list four most dominant kinetic energy Fourier modes. The entropy and the kinetic energy of these modes, $(k_x, k_z)$ in 2D and $(k_x, 0, k_z)$ in 3D, are very close, where $(k_x, k_y, k_z) = (\frac{\pi}{\sqrt{2}} n_x, \frac{\pi}{\sqrt{2}} n_y, \pi n_z)$. This is due the similarities between 2D and 3D RBC.

Using DNS, we studied the energy and entropy spectra and fluxes in 2D and 3D RBC for very large Prandtl numbers, and observed that they show similar scaling for large and infinite Prandtl numbers. The kinetic energy spectrum $E_u(k)$ scales as $k^{-13/3}$, while the entropy spectrum exhibits a dual branch, with a dominant $k^{-2}$ branch corresponding to the $\hat{\theta}(0, 0, 2n)$ Fourier modes.

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