STABILITY AND EXACT COHERENT STRUCTURES OF THE ASYMPTOTIC SUCTION BOUNDARY LAYER WITH TEMPERATURE GRADIENT

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Abstract The asymptotic suction boundary layer with a temperature gradient is a good point of entry to study the dynamics of thermal boundary layers by means of dynamical systems theory. The laminar flow without heating is parallel and its properties have been studied before. We add a temperature difference between the bottom plate and the free stream flow, and study the stability in dependence on Reynolds, Rayleigh and Prandtl number. In marked contrast to the usual Rayleigh-Bénard problem, the onset of convection is subcritical. Tracking secondary bifurcations we identify time-periodic, spanwise, and doubly-localized exact coherent states for this flow.

INTRODUCTION

Within the last two decades the use of dynamical system theory has enormously improved our understanding of the turbulence transition in flows without linear instabilities. Especially the study of exact coherent structures as an organizing element for the turbulent dynamics forms contributed to this progress [1, 2]. There is also evidence that coherent structures are relevant for fully turbulent flows [3]. As a step towards the study of spatially developing boundary layers, investigations of the parallel asymptotic suction boundary layer (ASBL) have revealed exact coherent structures [4, 5, 6] that could be relevant to the Blasius boundary layer as well.
Continuing this route of investigation, the ASBL with an uniformly heated plate might provide insights into the dynamics of thermal boundary layers. For this flow the laminar velocity profile as well as the laminar temperature profile have an exponential shape. Its can easily be explored using *Channelflow*-code [7] which we modified to include an additional temperature field.

STABILITY OF THE LAMINAR STATE

A stability analysis of the laminar profile of ASBL with a uniformly heated plate shows that the critical wave- and Rayleigh numbers depend strongly on the Prandtl number. E.g for $Pr = 1.0$ the laminar state becomes unstable at $Ra_{crit} = 19.48$ for a critical wavenumber $k_{crit} = 0.4305$. (The low values of the critical Rayleigh number are a consequence of the choice of the length scale.) For large values of Pr the critical wavenumber as well as the critical Rayleigh number move to higher values while for smaller Pr they move to lower values. Stability curves for $Re = 0$ and various values of the Prandtl number are shown in figure 1. For non-vanishing Reynolds number longitudinal rolls are the preferred instability for $Re < 54430$ while transversal rolls are preferred for $Re > 54430$, consistent with the onset of the TS-instability.

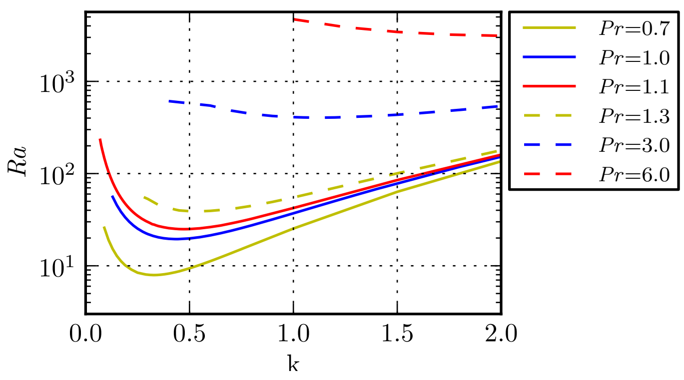

Figure 1. Stability curve of the laminar state for $Re = 0$ and different values of the Prandtl number.

EXACT COHERENT STRUCTURES

In all cases, even for $Re = 0$, the laminar state becomes unstable in a subcritical bifurcation. A visualization of the bifurcating two-dimensional roll solution for $k = k_{crit}$, $Ra = 13$ and $Pr = 1$ is shown in figure 2a). We track this roll solution in Rayleigh number to identify the position of the turning point and to study its dependence on the wavenumber. The lowest Rayleigh numbers for the turning point are found for wavenumbers smaller than the critical one. Furthermore, the upper branch was identified and we found that its velocity field extends much farther into the free stream than the one of the lower branch. A stability analysis reveals many secondary instabilities, including various long-wavelength instabilities that create solutions localized in one direction parallel to the plate. Examples of such localized solutions are shown in figure 2b) and c). In addition to these localized solutions we were also able to identify exact coherent structures that are localized in both directions parallel to the plate, as well as time-periodic solutions bifurcating from the roll solutions.

CONCLUSIONS

Our results show that the presence of a cross flow turns the subcritical thermal instability into a supercitical one.
The identification of exact coherent structures in a thermal boundary layer offers new ways to study the dynamics inside this boundary layer. Especially, the identification of localized structures might help to understand the dynamics of single thermal plumes.

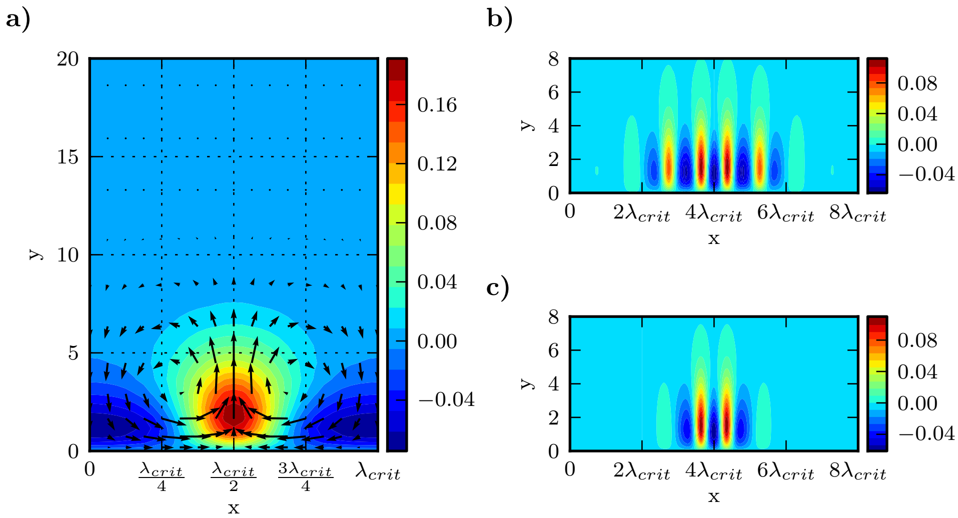

Figure 2. Flow- and temperature fields for the lower branch of the bifurcating roll-solution are shown in a). The flowfield is visualized by the arrows and the temperature is color coded. The Rayleigh number is 13, the Prandtl number is 1.0 and the spanwise wavelength is $k_{crit} = 0.4305$. In b) and c) the temperature field for two different spatially localized solutions is shown. In both cases the Rayleigh number is 17.8.

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