SPATIALLY-LOCALIZED TIME DEPENDENT SOLUTIONS INCLUDING TURBULENCE AND THEIR INTERACTIONS IN 2D KOLMOGOROV FLOW

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Abstract In 2D Kolmogorov flow in small aspect ratio domains, spatially-localized solutions such as kink, traveling or time-dependent kink-antikink pars coexist. However, the conservation of the flow rate in the y direction strongly restrict combination of localized solutions and their positioning. We find that by adding a homogeneous flow $U_y\hat{\boldsymbol{y}}$ their positioning is controlled and each of localized solutions including a spatially-localized chaos is isolated. Numerical results suggest that these isolated solutions can be elements constructing a whole flow.

INTRODUCTION

Spatially-localized turbulent states such as puffs and slags are commonly observed in transient flow or flows around at (nonlinear) critical Reynolds numbers. We expect that even such spatially-localized turbulence would be explained in terms of the dynamical system's approach that has successfully applied to "minimal" channel flows. However, there is still a long way because we have not obtained general means to describe spatially inhomogeneous or localized dynamics or solutions that might be embeded in the phase space we focus on.

As a first step, we adopt 2D Kolmogorov flow in small aspect ratio domains, and try to isolate each of spatially-localized solutions including spatio-temporal chaotic flows observed there by controlling the flow rate in y direction. 2D Kolmogorov flow in small aspect ratio domains ($\alpha = L_y/L_x$) for moderate Reynolds numbers, vorticity concentrates on a nallow area and localized kink-like structures emerge [1]. These kinks and anti-kinks can be elementary solutions describing a whole flow but their positioning is strongly constrained by the conservation of the flow rate in y direction.

We show that this conservation is controlled by adding a homogeneous flow U_y in the y direction and for some U_y a single localized structure consisting of a kink-antikink pair and spatially-localized chaos exist. Additionally, we confirm confirm that the tail or the remaining parts of the localised solution has little dependance on U_y. This independence suggests that such localized solutions can coexist with other localized solutions belonging to other U_y. With this fact we expect that a whole flow should be described as a combination of localized solutions obtained for U_y.

PHENOMENOLOGY OF LOCAL STRUCTURE

Recent works on a transient state where turbulence and laminar flow coexist and emphasis on elemental dynamics motivates us to search a solution which is spatially-localized and has the translational symmetry in x direction ($\mathcal{T}$-symmetry) in the other part of the domain. However, this situation never realize. Since boundary conditions are periodic, $\int dx u_y = \int dx \partial_x \psi = \psi(L) - \psi(0) = 0$. We assume that the width of a localized structure is small enough compared with L_x and set it $\epsilon << L_x$. The width of the tail part with $u_y = u_i$ is denoted by L_i. Then, $\Sigma u_{yi} L_i + O(\epsilon) = 0$. Especially, if $|u_i| = u > 0$ in any domains between localized structures, the length L_p with positive u_y and the length L_n with negative u_y satisfy,

$$L_p - L_n + O(\epsilon) = 0 \tag{1}$$

In this situation, a localized solution in a simply-connected domain corresponds to $L_p \sim L_x$ and $L_n \sim 0$, which unable to satisfy (1). Equation (1) rather means $L_p \sim L_n \sim L_x/2$.

In order to relax the condition (1), the velocity $\boldsymbol{u}$ is divided into a constant velocity $\boldsymbol{U}$ and the remaining part $\boldsymbol{v}$ and the equations for $\boldsymbol{v}$ are solved under the periodic boundary conditions. We consider when $\boldsymbol{U} = U_y\hat{\boldsymbol{y}}$. Then, the counterpart of Eq.(1) is,

$$L_p = \frac{L_x}{2}(1 + \frac{U_y}{u}) + O(\epsilon) \tag{2}$$

,

$$L_n = \frac{L_x}{2}(1 - \frac{U_y}{u}) + O(\epsilon) \tag{3}$$

. U_y controls the degree of the asymmetry of L_p and L_n. Note that $U_y = 0$ leads to the original situation. By controlling U_y we can realize $L_p \sim L_x$ and $L_n \sim 0$.

NUMERICAL METHOD

We deal with the following non-dimensionalized vorticity equation for $\omega = \partial_x v_y - \partial_y v_x$ and $Re = \frac{\sqrt{\chi}}{\nu}(\frac{L_y}{2\pi})^{3/2}$

$$\partial_t \omega + v_x \partial_x \omega + (v_y + U_y)\partial_y \omega = \frac{1}{Re}\nabla^2 \omega - n\cos(ny) \qquad (x, y) \in [0, 2\pi/\alpha] \times [0, 2\pi] \tag{4}$$

Here, μ the kinematic viscosity, n the force wave number and χ the forcing amplitude. This equation is solved using Fourier-Fourier pseudospectral method and 4th order Runge-kutta method. Numerical resolutions used were 128 modes per 2π and aliasing errors are removed using the two-thirds rule. Time step is $dt = 0.001$.

NUMERICAL RESULTS

Figure 1 shows a snapshot of spatially-localized turbulence obtained for Re = 50, $\alpha = 0.25$ and $U_y = 2.0$. The width of disordered domain is kept nearly constant (Figure 2). Figure 3 shows snapshots of a recurrent solution (snake) and a traveling kink-antikink pair for Re = 20 and $\alpha = 0.25$. The traveling wave solutions are confirmed to exist for $U_y \in [0.46, 0.64]$. All of these solutions have almost the same tail part (marginal flow). Superposing a traveling wave solution and a recurrent standing solution we examine the collision of the localized solutions. As shown in Fig.4, the traveling wave collides with the standing solution and then passes through the latter with some phase shift.

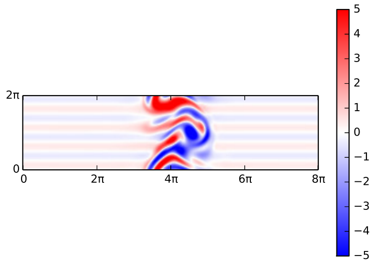

Figure 1. A snap shot of vorticity field for localized turbulence at $Re = 50$ $\alpha = 0.25$ $U_y = 2.0$.

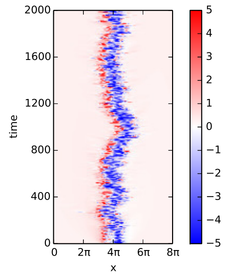

Figure 2. The evolution of the cross-section at $y = \pi/4$ of the vorticity of the localized turbulence shown in Fig.1.

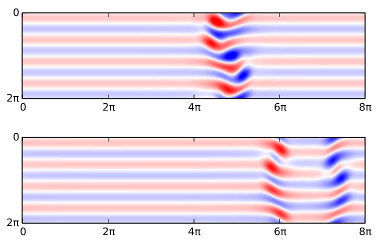

Figure 3. Localized solution for $Re = 20$ $\alpha = 0.25$.The upper panel shows a recurrent solution for $U_y = 0.80$. The lower panel shows a traveling wave solution for $U_y = 0.55$

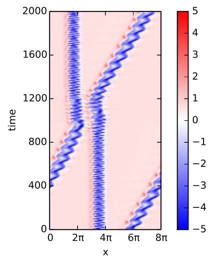

Figure 4. The evolution of the cross-section at $y = \pi/4$ of the vorticity field for $Re = 20$, $\alpha = 20$ and $U_y = 0.60$. A traveling wave solution colides with a recurrent solution.

SUMMARY

We find that the conservation of the flow rate in the y direction can be controlled by adding a homogenious flow. The phenomenology on the positioning of localized structures in 2D Kolmogorov flow suggests spatially-localized solutions can be obtained by controlling U_y. Using direct numerical simulations, such localized solutions are confirmed to be exist. In the talk, we will report the detail of the interactions among localized solutions including localized turbulence and also discuss the possibility of the description of a whole flow with the localized solutions.

References

[1] Dan Lucas and Rich Kerswell. Spatiotemporal dynamics in two-dimensional Kolmogorov flow over large domains. *Journal of Fluid Mechanics*, **750**:518–554, June 2014.