PROPERTIES OF MAGNETIC ENERGY AND MAGNETIC HELICITY CASCADES IN MHD TURBULENCE

Stepanov Rodion[1,2], Frick Peter [1] & Mizeva Irina [1]
[1]*Institute of Continues Media Mechanics, Perm, Russua*
[2]*Perm National Research Polytechnic University, Perm, Russia*

Abstract Magnetohydrodynamic (MHD) turbulence is an important part of astrophysical processes, which gives rise to global cosmic magnetic fields. Over the last few decades, the peculiarities of MHD turbulence have attracted the interest of researchers in astrophysics and fluid dynamics, significant attention has been paid to the role of magnetic helicity in fully developed MHD turbulence. Magnetic helicity, together with the energy and cross-helicity, is one of the three integrals of motion in ideal MHD. We show that oppositely directed fluxes of energy and magnetic helicity coexist in the inertial range in fully developed magnetohydrodynamic (MHD) turbulence with small-scale sources of magnetic helicity. Using a helical shell model of MHD turbulence, we study the high Reynolds number magnetohydrodynamic turbulence with well separated scales of energy input, magnetic helicity input and magnetic helicity sink. We obtain three inertial ranges with different scaling properties. In a short range of scales larger than the forcing scale of magnetic helicity, a bottleneck-like effect appears, which results in a local reduction of the spectral slope. The slope changes in a domain with a high level of relative magnetic helicity, which determines that part of the magnetic energy related to the helical modes at a given scale. In the infrared part of the spectra we observe simultaneous inverse cascade of energy and magnetic helicity. Our results indicate that a large-scale dynamo can be affected by the magnetic helicity generated at small scales. The kinetic helicity, in particular, is not involved in the process at all.

SHELL MODEL

Direct numerical simulations (DNS) of statistically stationary turbulence with a substantially helical magnetic field are complicated because they require adequate separation of the forcing scale and dissipation scale for the energy and magnetic helicity. An attempt at this kind of simulations was performed by [1], who showed that the inverse cascade of the magnetic energy and helicity is provided by local interactions during turbulence development and by non-local interactions in the saturated state. However, in this model, the spectral fluxes of magnetic helicity and energy were not separated. The significant direct cascade of magnetic helicity obtained can be explained by the proximity of the dissipation scale to the forcing scale.

Here, we try to highlight the role of magnetic helicity by separating its source from the source of energy. The MHD turbulence at large Reynolds numbers can be simulated with a help of the shell models governed by equations

$$
\begin{aligned}
d_t U_n &= \tilde{W}_n(U,U) - \tilde{W}_n(B,B) - \frac{U_n^2}{Ru} + F_n^U \\
d_t B_n &= \tilde{W}_n(U,B) - \tilde{W}_n(B,U) - \frac{B_n^2}{Rm} + F_n^h + F_n^d .
\end{aligned}
\qquad (1)
$$

Shell models are low-dimensional dynamic systems that are derived from the original MHD equations by a drastic reduction of the number of variables. These models describe the dynamics and spectral distributions of fully developed MHD turbulence through a set of complex variables U_n, B_n, which characterize the kinetic energy $E_n^u = |U_n|^2/2$ and magnetic energy $E_n^b = |B_n|^2/2$ of pulsations in the wave number range $k_n < |\mathbf{k}| < k_{n+1}$ (called the shell n), where $k_n = \lambda^n$ (λ is the shell width in a logarithmic scale, typically chosen to be equal 1.618).

For numerical simulations of MHD turbulence we used the shell model of MHD turbulence described in [3] with the following parameters: $Ru = Rm = 10^6$. In the non-dissipative limit, equations conserve the total energy $E = \sum(E_n^u + E_n^b)$, the cross-helicity $H^c = \sum(U_n B_n^* + B_n U_n^*)/2$ and the magnetic helicity $H^b = \sum \imath k_n^{-1}((B_n^*)^2 - B_n^2)/2$. We separated scales of energy input ($k = 1$), magnetic helicity ($k = 100$) and sink of magnetic helicity together with magnetic energy ($k = 0.1$).

We consider MHD turbulence that is stationary forced at the largest scale, with a source of magnetic helicity that is localized at a scale inside the pronounced inertial range, in the manner as it is described in [2]. In our research, we consider an infrared part of the spectra in order to focus on the possibility of a simultaneous direct cascade of energy to the small scales and oncoming inverse cascade of magnetic energy and magnetic helicity to the large scales from the forcing scale. Adding an ad hoc dissipation term at large scale helps to achieve a statistically stationary state.

RESULTS

In Fig. 1a we show the compensated spectrum of total energy ($U_n^2 + B_n^2$), in which three different ranges can be separated in inertial scales. In the range $0.1 < k < 1$ an inverse cascade of magnetic helicity is observed, which transfer a small

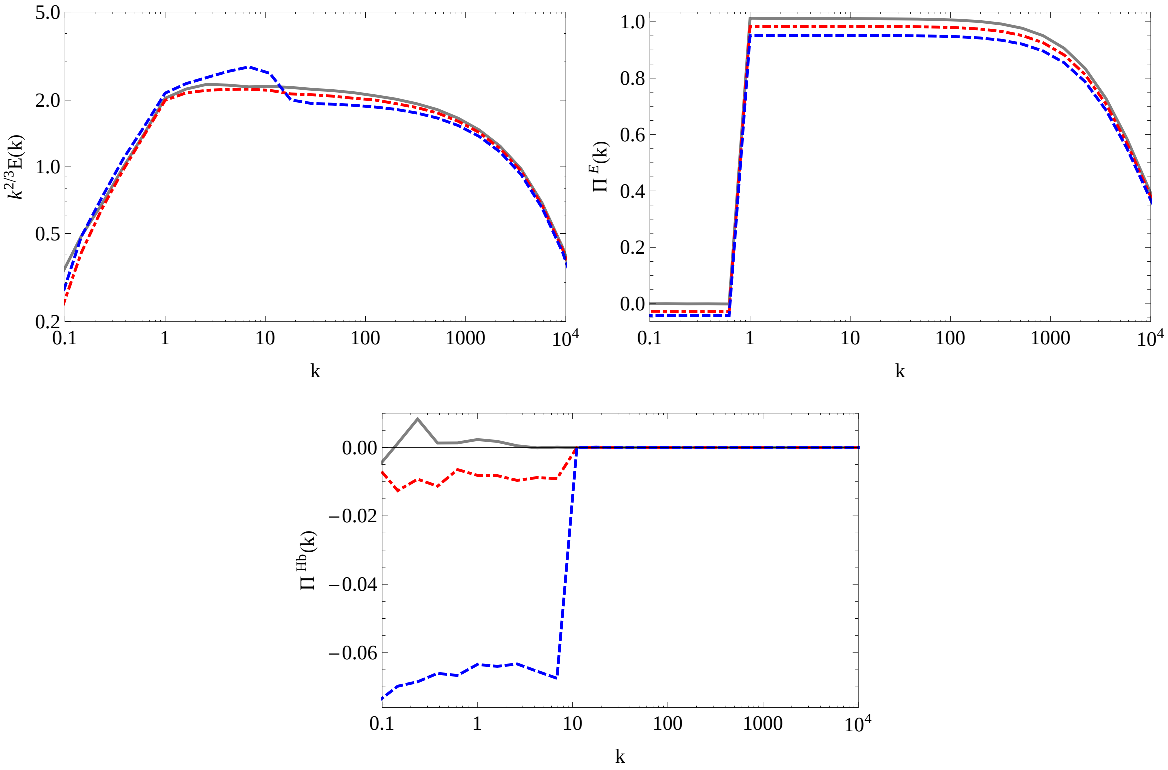

Figure 1. (a - Spectral power density of MHD turbulence in case of f=1, d=1, h=0.1, (red solid line), h=1 (dashed line), h=10 (dotted line) (averaging 2000 time units), b - total energy flux, c - magnetic helicity flux

part of magnetic energy to large scales. There is an energy equipartition in this range of wave numbers. In the range $1 < k < 10$ an inverse cascade of magnetic helicity coexists with the direct cascade of energy (Fig. 1b), which leads to decrease of the spectral slope and appearance of a pileup in the spectra on the scale of magnetic helicity injection. This bottle-neck like effect is described in details in [3]. Magnetic helicity effects only in scales larger than the scale of its input. That's why we observe Kolmogorov spectra at $k > 10$.

CONCLUSION

In fully developed MHD turbulence, a source of magnetic helicity at small scale provides a negative spectral flux, which coexists with the direct energy flux in the inertial range. Near the scale of helicity injection a bottleneck-like effect appears, which leads to a local reduction of the spectral slope. We found that the key quantity for understanding this effect is the magneticto- magnetic energy spectral flux. This flux, being negligible in the non-helical case, is negative and clearly associated with an inverse cascade of magnetic helicity independent of the sign of the injected helicity. The same effect can be obtained even for an alternating source of small-scale magnetic helicity with a zero-mean injection rate. Magnetic helicity helps to transfer the magnetic energy from forcing scale to the largest scale of the system. In spite of the rather special conditions in our modelling, a similar scenario to some extend can develop in realistic situations, e.g., magnetic helicity injection into the corona in emerging active regions [4].

Acknowledgments This work benefited from the Russian Foundation of Basic Research grant 14-01-96010. Computing resources of the supercomputer URAN were provided by the Institute of Mathematics and Mechanics UrB RAS.

References

[1] Alexakis, A., Mininni, P. D., & Pouquet, A. 2006, ApJ, 640, 335
[2] Plunian, F., Stepanov, R., & Frick, P. 2013, PhR, 523, 1
[3] Stepanov R., Frick P., & Mizeva I. 2015, ApJ, 798, L35
[4] Liu, Y., Hoeksema, J. T., Bobra, M., et al. 2014, ApJ, 785, 13