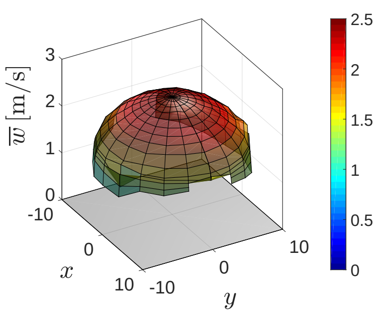
# RECONSTRUCTION OF TURBULENT PIPE-FLOW PROFILES FROM LASER DOPPLER VELOCIMETRY DATA

Denis F. Hinz[1]
[1]Kamstrup A/S, Industrivej 28, Stilling, 8660 Skanderborg, Denmark.

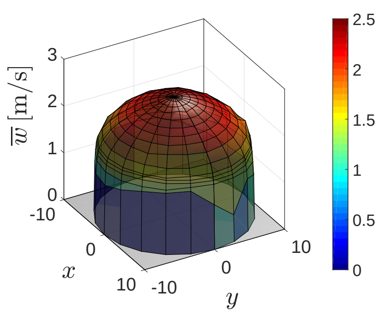
*Abstract* Laser Doppler velocimetry (LDV) is a powerful experimental tool that finds increasing application in industrial research and product development. For example, LDV may be used to support the development of water, heat, and cooling meters as well as the design and validation of the associated testing and calibration facilities. For such applications, a basic quantity of interest is the volume flow rate $Q$ that can be determined through numerical integration of the velocity profile over the measurement grid. However, under realistic experimental conditions, optical disturbances and reflections may cause corrupted and unreliable data at various measurement points. Here, we study a criterium to identify and reconstruct such data points. The reconstruction criterium is based on the standard error associated with each measurement point. Using some of our LDV data, we show that the error in $Q$ with respect to a reference flow rate $Q_{\mathrm{ref}}$ may be reduced significantly through the reconstruction. Consequently, the reconstruction criterium allows to establish accurate and reliable flow rates that have potential to be used as a primary validation reference.

## Introduction and experimental setup

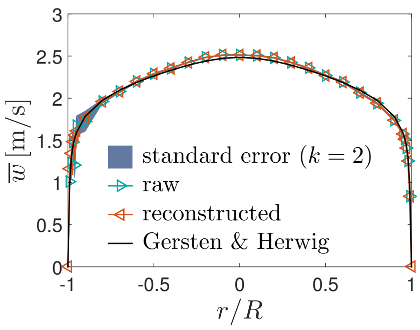
We use a commercial LDV system (Optolution/ILA) where the optical access is realized through a glass pipe inside a window chamber that allows the full 3D measurement of pipe-flow profiles. LDV measurements of the axial component $w$ are conducted on a measurement grid of 261 points in a polar coordinate system. In Fig. 1 (a) we show the mean axial component over the cross-sectional coordinates from a representative example data set of fully developed turbulent pipe flow that was measured on a calibration flow bench and contains various missing and corrupted (i.e. unreliable) data points. Consequently, an integration of the raw data results in significant errors in $Q$. Here, we investigate criteria to identify missing and unreliable data points and reconstruct the velocity through interpolation of data from neighboring measurement points. The reconstructed velocity profiles (Fig. 1 (b)) are integrated numerically to estimate $Q$.



(a) Raw velocity profile with missing and corrupted data points.



(b) Reconstructed velocity profile.




(c) Representative comparison of one raw and reconstructed velocity profile ($\theta = 108°$) along with a theoretical reference profile (Gersten [2])

Figure 1: Reconstruction of the mean velocity profile $\overline{w}$. $\mathrm{Re} = 4.0 \cdot 10^4$, $T = 20°$ C.

## Reconstruction method

For grid measurements of $w$, the amount of data acquired at each measurement point is determined by the choice of two experimental constraints: (I) the maximal number of samples $n_{\max}$ and (II) a timeout $t_{\max}$ for each point measurement. For the present measurements we choose $n_{\max} = 1000$ and $t_{\max} = 60\,\mathrm{s}$. From the velocity samples $w_i$ we determine

$$\overline{w} = \frac{1}{n}\sum_{i=1}^{n} w_i \quad \text{and} \quad \mathrm{Tu} = \frac{\sigma_w}{\overline{w}}, \tag{1}$$

where $\overline{w}$ is the estimator for the mean velocity and Tu is the turbulence intensity with $\sigma_w$ the standard deviation of the samples $w_i$. To estimate the reliability of the estimator $(1)_1$ at each spatial measurement point, we determine the associated standard error

$$\sigma_{\overline{w}} = \frac{\sigma_w}{\sqrt{n}}. \tag{2}$$

The standard error (2) provides an uncertainty estimate for the estimator of the mean velocity $(1)_1$. In view of (1) and (2), the relative standard error is directly related to the turbulence intensity and the number of samples through

$$\frac{\sigma_{\overline{w}}}{\overline{w}} = \frac{\mathrm{Tu}}{\sqrt{n}}. \tag{3}$$

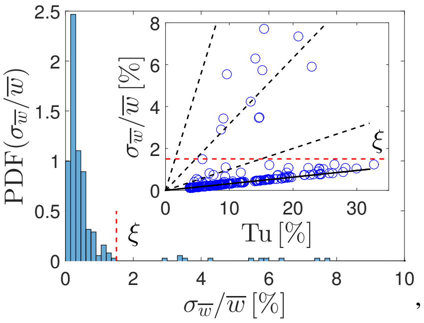
Hence, (3) is as natural choice for defining reconstruction criteria based on the reliability and we choose a simple criterium

$$\frac{\sigma_{\overline{w}}}{\overline{w}} > \xi, \tag{4}$$

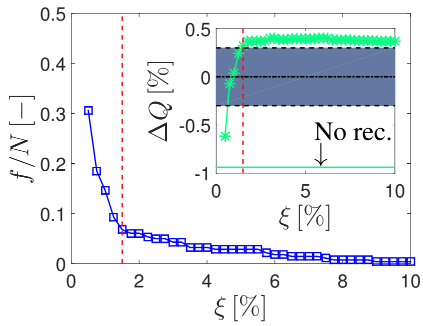
where $\xi$ is the threshold value for admissible standard errors. Points that fulfill the criterium (4) are reconstructed through interpolation from neighboring points. Notice that there are two special cases: for $n = 1$ (only one sample) and $n = 0$ (no sample) the standard error (2) is degenerate. Points that fulfill these special cases are also reconstructed.

**Results and discussion**

Inspection of the relative standard errors through a scatter plot of $\sigma_{\overline{w}}/\overline{w}$ versus $\mathrm{Tu}$ and the probability density function (PDF) of $\sigma_{\overline{w}}/\overline{w}$ shows that the majority of measurement points exhibit standard errors $\lesssim 1.0\,\%$ (Fig. 2 (a)). To assess the performance of the reconstruction regarding the estimation of the integrated flow rate $Q$, we estimate the error $\Delta Q = (Q - Q_{\mathrm{ref}})/Q_{\mathrm{ref}}$ with respect to a reference flow rate $Q_{\mathrm{ref}}$ for different choices of $\xi$. $Q_{\mathrm{ref}}$ is obtained from a magneto-inductive (MID) master meter that is calibrated against weight with an expanded uncertainty of $U = 0.3\,\%$ ($k = 2.0$). Without reconstruction, $\Delta Q = -0.94\,\%$ (solid green line in inset of Fig. 2 (b)). For the weakest reconstruction criterium (where only points with $n = 0$ and $n = 1$ are reconstructed), the error decreases to $\Delta Q = 0.37\,\%$ (inset of Fig. 2 (b)). For the choice of $\xi = 1.5\,\%$, the error is $\Delta Q = 0.32\,\%$, which represents a slight improvement over the weakest reconstruction criterium and is close to the expanded uncertainty associated with the reference meter. Technically, a further decrease in $\Delta Q$ may be achieved by choosing $\xi$ below $1.5\,\%$. However, the ratio $f/N$ of the number of reconstructed points $f$ to the total number of measurement points $N$ increases drastically, indicating that a large amount of points is reconstructed, which results in physically unreasonable profiles. The PDF (Fig. 2 (a)) confirms this relationship, since lower values of $\xi$ result in significant cutoffs in the PDF.




(a) Illustration of reconstruction criteria.




(b) Impact of choice of $\xi$ on $\Delta Q$ and $f/n$.

Figure 2: The optimal choice of $\xi$ is found to be $\xi \approx 1.5\,\%$: For this choice of $\xi$, the estimated error is $\Delta Q = 0.32\,\%$, whereas the raw data without reconstruction exhibits $\Delta Q = -0.94\,\%$. The shaded area in the inset of (b) indicates the expanded calibration uncertainty associated with $Q_{\mathrm{ref}}$.

**Conclusion and outlook**

A simple and objective reconstruction method based on a criterium for the standard error at each measurement point is studied. The reconstruction yields significant improvements in the error $\Delta Q$ and reduces its value close to the calibration uncertainty $U = 0.3\,\%$ ($k = 2.0$) of the master meter. The reconstruction allows to establish accurate and reliable flow rates that have potential to be used as a primary validation reference. Modifications in the reconstruction criterium and the integration method as well as improved boundary layer approximations have not been studied yet, but might well yield further improvements (cf., e.g. Büker [1]).

**References**


[1] Oliver Büker. *Untersuchungen zur Darstellung und Weitergabe der Skala 'Volumen von Wasser' mithilfe laseroptischer und konventioneller Messverfahren*. PhD thesis, 2010.

[2] Klaus Gersten. Fully developed turbulent pipe flow. In W. Menzenkirch, editor, *Fluid Mechanics of Flow Metering*, pages 1–22. Springer, second edition, 2005.