
ENERGY FLUX IN ISOTROPIC TURBULENCE UNDER LARGE VARIATIONS OF EXTERNAL FORCING

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Abstract We investigate the response of energy flux in isotropic turbulence to step-function like perturbation in external forcing at large length scales. From both physical experiments and direct numerical simulations, we measured the evolution of the Eulerian velocity structure functions, such as $D_{LL}(r)$, $D_{NN}(r)$, before and after the perturbation in forcing. In both cases, we observed the cascade of the energy excess at large scales cascade through scales to the dissipative range, which can be used to study the dynamics of the cascade, and in particular, to estimate the relevant time scales.

INTRODUCTION

Most of studies on turbulence are on statistically stationary turbulence, in which the mean rate of energy injection into the flow is balanced exactly by the mean energy dissipation rate. On the contrary, for non-stationary turbulence, such a balance is violated. For example, the energy injection rate is zero in the decaying turbulence behind a grid [1, 12, 6]. In the presence of a strong homogeneous shear [13, 11], a fast growth of turbulence energy with significant anisotropy is observed, describable in terms of rapid distortion [5]. The energy input can also be isotropic but periodic in time, as it happens when studying turbulence under modulation [8, 15, 16, 2, 7, 3]. Much insight on turbulence can be gained from investigations on these non-stationary flows. Even in stationary turbulence the instantaneous energy injection rate is fluctuating and it has been observed in numerical simulations [11, 10] that the peaks of instantaneous energy dissipation rate lag behind the peaks of instantaneous energy injection rate. The response of turbulence to unsteady energy injection can therefore reveal the turbulent energy cascade process.

In this work, we are focusing on isotropic turbulence with varying forcing. Previous studies on modulated turbulence [15, 16, 2, 7, 3] focused on the energy dissipation alone. Very recently, the energy cascade through scales was studied experimentally [4] and numerically [14]. Here we present results from an experimental and numerical study of an isotropic turbulent flow under a step-function like perturbation of the large-scale forcing. We explore the response of the energy cascade by following the evolution the structure functions, such as the second-order velocity structure functions $D_{LL}(r)$ and $D_{NN}(r)$.

EXPERIMENTAL SETUP

The turbulent water flow is generated in the Lagrangian Exploration Module (LEM), an icosahedron shaped container with 12 independently driven propellers, one on each vertex [17, 18]. The edge length of the icosahedron is 40 cm and the apparatus contains 140 liters of water. Schematically, the turbulence in the LEM can be thought of as a superposition of six von Kármán swirling flows, each created by a pair of counter-rotating propellers. The icosahedron geometry gives a spatially symmetric arrangement of the axes of these six von Kármán flows in order to produce highly isotropic turbulence at the center. It has been shown [18] that when the propellers are rotating at the same frequency, the turbulence in the center region of the LEM (approximately 15 cm in diameter) is nearly homogeneous and isotropic. As the propeller speed increases, the energy injection rate into the flow increases and the Reynolds number of the flow increases accordingly.

In our experiments we started with all 12 propellers rotating at 200 revolution per minute (rpm) until the flow was steady and then increased the speed of all propellers simultaneously to 400 rpm, corresponding to a change in R_λ from 260 to 350. We performed Lagrangian Particle Tracking (LPT) measurements from 5 seconds before to 40 seconds after the motor speed change. This recording time is more than 60 large-eddy turnover time of the steady-state turbulence at 400 rpm. We then repeated the experiment 400 times to have a large ensemble of independent realizations.

DIRECT NUMERICAL SIMULATIONS (DNS)

The DNS simulation was performed using a standard spectral code with specified energy injection rate in the lower modes. In this work, we used 768^3 modes in a periodic domain. The flow was first run to a steady state and maintained for a very long time. Then 9 independent events were chosen from the long record as the initial conditions. Starting from each of these initial conditions, the simulation was performed again for approximately 4 large-eddy turnover times with the energy injection rate increased to 8 times of the initial value.

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RESULTS

For the experimental data, we obtained velocities of the tracer particles from the measured three-dimensional particle trajectories by differentiation in time. The Eulerian structure functions were then obtained from these velocity data because several hundreds of tracers were followed at any given time. For the DNS data, the structure functions were computed from the energy spectra [9].

Figure 1 shows the second order longitudinal velocity structure function $D_{LL}(r)$ at several times, where $t = 0$ corresponds to the time of the change of external forcing. (For experimental data, the time was shifted because of the time needed for the energetic fluids agitated by the propellers to arrive at the center of the LEM, where the measurements were performed.) In both cases, it can be seen clearly that the energy excess is initially at the large scales and then propagates down to smaller and smaller scales. This process resembles the energy cascade of a statistically stationary state.

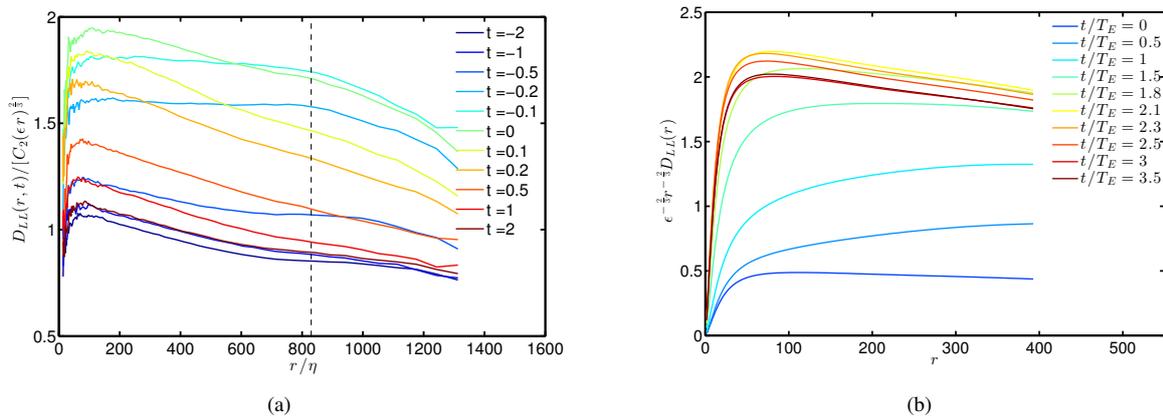


Figure 1. The evolution of $D_{LL,r}$ at several times, with $t = 0$ corresponding to the time of the change of external forcing. (a) Data from experiments; (b) Data from DNS.

Figure 1 also reveals that the level of energy at small scales overshoots, before reaching the steady-state value that is observable asymptotically at long times. This behavior, seen both experimentally and numerically, suggests a richer dynamics than what anticipated from previous theoretical studies, and points to interesting further research directions.

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