ON THE ROLE OF THE HELICITY IN THE ENERGY TRANSFER IN THREE-DIMENSIONAL TURBULENCE

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Abstract
Behavior of the turbulent flows could be changed by changing the nature of the external force or the confining geometry which essentially results in breaking some of the symmetries of the ideal homogeneous and isotropic flows. In a numerical simulation, however, it is possible to selectively break symmetries of the Navier-Stokes equations with other constraints like helicity. In a recent [1] simulation of a decimated version of the incompressible three dimensional Navier-Stokes equations, where helicity was maintained sign-definite using a helical projection, a reversal of energy cascade similar to two-dimensional Navier-Stokes equations was observed. The sign-definite helicity breaks the parity symmetry of the flow. It is one of the important symmetries of the flow that contributes to the forward energy cascade in three dimensional Navier-Stokes equations. In our study we measure the degree to which the parity symmetry controls the direction of the cascade. We introduce a mechanism in which the parity is broken stochastically but in a time frozen manner with helical constraints. We keep triadic interactions in Fourier space involving modes with definite sign of helicity and decimate the triads of other modes with opposite sign of helicity with a fixed probability. We studied the cascade of energy in three dimensional turbulence by changing the relative weight between positive and negative helicity modes. We present the results from our recent simulations.

INTRODUCTION

Numerous experiments in the three-dimensional homogeneous and isotropic turbulence and direct numerical simulations (DNS) of the three-dimensional Navier-Stokes equations for incompressible flows show cascade of energy to the small scales when forced at the large scales. Two of the invariants of three-dimensional Navier-Stokes equations are the energy $E = \int d^3r\ \vec{u}\cdot\vec{u}$ and the helicity $H = \int d^3r\ \vec{u}\cdot\vec{\omega}$, where $\vec{u}$ is the velocity and $\vec{\omega} = \nabla\times\vec{u}$ is the vorticity. The energy is positive and definite whereas the helicity could be either positive or negative. Both energy and helicity cascade forward from large scales to small scales [2]. The Energy spectrum $E(k) = \sum_{\boldsymbol{k}\ni|\boldsymbol{k}|=k} |\mathbf{u}(\boldsymbol{k})|^2$ shows a Kolmogorov $k^{-5/3}$ scaling in the inertial range.
However there are evidences of energy transfer to the large scales under special conditions. A turbulent flow confined in thick fluid layers due to formation of large scale vortex suppresses vertical motions and supports large scale energy transfer [3]. In a rotational turbulent flow with helical force a direct cascade of helicity and direct and inverse cascade of enegy is observed [4]. Positive definiteness of helicity leads to inverse energy tranfer. Dynamics of the inverse energy transfer is studied in a subset of all interactions in the NS equations as in Fig.1a. There is a growth of helicity at the small scales, both in positive and negative modes but remains finite because of the mirror symmetry.
In this contribution we change the relative weight between the positive and negative helicity modes with a control parameter using a method to separate positive and negative modes to understand the dynamics of the energy cascade in particular to study the transition of forward energy transfer to the inverse energy transfer.

NUMERICAL METHOD

In Fourier space, $\boldsymbol{u}(\boldsymbol{k},t)$ has two degrees of freedom since $\boldsymbol{k}\cdot\boldsymbol{u}(\boldsymbol{k},t) = 0$. We chose these two degrees to be the projections on orthonormal helical waves with definite sign of helicty [5]; $\boldsymbol{u}(\boldsymbol{k},t) = a^+(\boldsymbol{k},t)\boldsymbol{h}^+(\boldsymbol{k}) + a^-(\boldsymbol{k},t)\boldsymbol{h}^-(\boldsymbol{k})$, where $\boldsymbol{h}^\pm(\boldsymbol{k})$ are the complex eigenvectors of the curl operator $i\boldsymbol{k}\times\boldsymbol{h}^\pm(\boldsymbol{k}) = \pm k\boldsymbol{h}^\pm(\boldsymbol{k})$. We define a projection operator $\mathcal{P}^\pm(\boldsymbol{k}) \equiv \frac{\boldsymbol{h}^\pm(\boldsymbol{k})\otimes\boldsymbol{h}^\pm(\boldsymbol{k})^*}{\boldsymbol{h}^\pm(\boldsymbol{k})^*\cdot\boldsymbol{h}^\pm(\boldsymbol{k})}$ which gives us $\boldsymbol{u}^\pm(\boldsymbol{k},t) = \mathcal{P}^\pm(\boldsymbol{k})\boldsymbol{u}(\boldsymbol{k},t)$ and the velocity is then could be decomposed as $\boldsymbol{u}(\boldsymbol{k},t) = \boldsymbol{u}^+(\boldsymbol{k},t) + \boldsymbol{u}^-(\boldsymbol{k},t)$. We then write the eneregy as $E(t) = \sum_{\boldsymbol{k}} |\boldsymbol{u}^+(\boldsymbol{k},t)|^2 + |\boldsymbol{u}^-(\boldsymbol{k},t)|^2$. and the helicity as $\mathcal{H}(t) = \sum_{\boldsymbol{k}} k(|\boldsymbol{u}^+(\boldsymbol{k},t)|^2 - |\boldsymbol{u}^-(\boldsymbol{k},t)|^2)$. We could write the Navier-Stokes equations independently for each modes in the Fourier space as

$$\partial_t\boldsymbol{u}^\pm(\boldsymbol{k},t) = \mathcal{P}^\pm(\boldsymbol{k})\boldsymbol{N}_{u^\pm}(\boldsymbol{k},t) + \nu k^2\boldsymbol{u}^\pm(\boldsymbol{k},t) + \boldsymbol{f}^\pm(\boldsymbol{k},t) \tag{1}$$

where ν is the kinematic viscosity and $\boldsymbol{f}$ is the external forcing and the nonlinear term containing all triadic interactions is $\boldsymbol{N}_{u^\pm}(\boldsymbol{k},t) = \mathcal{FT}(\boldsymbol{u}^\pm\cdot\boldsymbol{\nabla}\boldsymbol{u}^\pm - \boldsymbol{\nabla}p)$.

DISCUSSION AND RESULTS

There are four classes of nonlinear triadic interactions with definite helicity signs under helical decomposition of NS equations. Energy and helicity are conserved for each triads. Full decimation of triads involving either $\boldsymbol{u}^+$ or $\boldsymbol{u}^-$ shows inverse cascade of energy [1, 6, 7] as shown in Fig.1b. To understand the transition of the forward energy transfer to

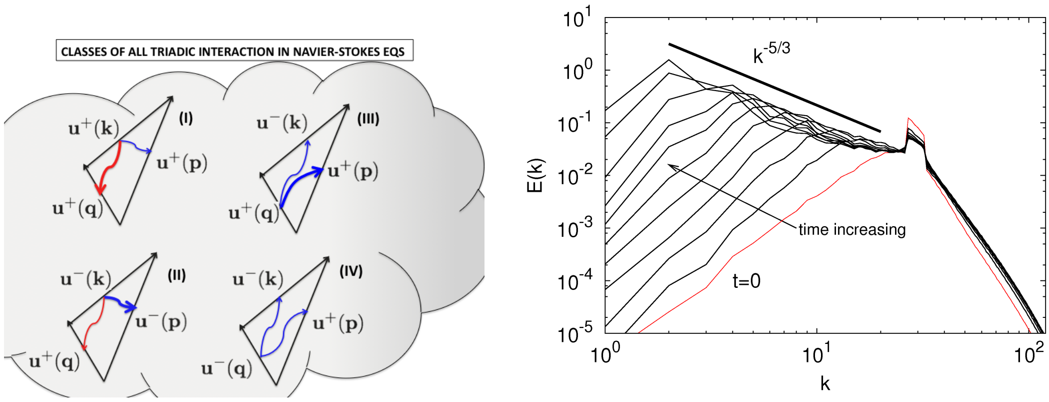

Figure 1. (a) All possible traidic interactions in Navier-Stoke's equations. The triads with only $\boldsymbol{u}^+$ lead to reversal of energy cascade (left). (b) Energy spectra in the inverse cascade regime shows $k^{-5/3}$ slope [1] (right).

the inverse energy transfer we decimated a fraction ϵ of modes with helicty of one sign instead of all of them. We numerically found a *critical value* of ϵ at which forward cascade of energy stops when force is applied at the large scales and alternatively, inverse cascade of energy stops if forced at small scales. We modified the projection operator using the parameter ϵ,

$$\mathcal{P}^+_\epsilon(\boldsymbol{k})\boldsymbol{u}(\boldsymbol{k},t) = \boldsymbol{u}^+(\boldsymbol{k},t) + \theta_\epsilon(\boldsymbol{k})\boldsymbol{u}^-(\boldsymbol{k},t) \tag{2}$$

where $\theta_\epsilon(\boldsymbol{k})$ is 0 or 1 with probability ϵ and $1-\epsilon$, respectively. When $\epsilon = 0$ we obtain the results for standard Navier-Stokes equations and when $\epsilon = 1$ we recover results for fully helical-decimated Navier-Stokes equations. We performed direct numerical simulations of Eq.(1) using a pseudo-spectral method on a periodic cubic domain of size $L = 2\pi$ with resolutions upto 512^3 collocation points using this projection operator for decimation.

We observe that as we increase ϵ, the contribution of triads leading to inverse energy cascade grows. Only when ϵ is very close to 1 inverse energy cascade takes over the forward cascade; critical value of ϵ is found to be $\simeq 1$ our simulations. We also measure the relative changes in the intermittency in the forward cascade regime at changing ϵ.

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