EFFECT OF VISCOELASTICITY ON THE FAR-DISSIPATION RANGE OF HOMOGENEOUS ISOTROPIC TURBULENCE

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Abstract The comparison of the results of direct numerical simulations of isotropic turbulence of Newtonian and viscoelastic fluid, based on the FENE-P model, provide evidence that viscoelasticity modifies qualitatively the behavior of the smallest scales: we observe a k^{-6} law in the far dissipation range of the energy spectrum and we show that it is a robust feature, roughly independent of the large-scale dynamics. It is further shown that the drag-reduction in such flows, as measured by the difference in energy dissipation between Newtonian and viscoelastic flow, strongly depends on the initial conditions.

INTRODUCTION

In this communication we will focus on two features of viscoelastic flows by considering the dynamics of the FENE-P model, representing some key features of flow with a weak concentration of polymer molecules. Firstly we will establish the robust presence of a steep power-law dependence of the energy spectrum in the far dissipation range (previously observed in [1]). Secondly we will investigate the drag reduction and in particular how initial conditions affect it.

MODEL AND NUMERICAL SET-UP

We have implemented the FENE-P model [2] in a standard pseudo-spectral solver. The model is given by

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \times (\nabla \times \boldsymbol{u}) = -\nabla p + \nu \Delta \boldsymbol{u} + \nu_p \,\mathrm{div}\,(P.\boldsymbol{C} - \boldsymbol{I}) + \boldsymbol{f} \quad (1)$$

$$\partial_t \boldsymbol{C} + \boldsymbol{u}.\nabla \boldsymbol{C} = \boldsymbol{C}.\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^t.\boldsymbol{C} + \frac{1}{\tau_p}(P.\boldsymbol{C} - \boldsymbol{I}) \quad (2)$$

with $p = p' + \frac{u^2}{2}$ the total pressure, $\boldsymbol{C}$ the conformation tensor of the polymer chains, $\boldsymbol{I}$ the identity tensor, $P = (L^2 - 1)/(L^2 - Tr(\boldsymbol{C}))$ the Peterlin's function with L the maximum extension of the spring, τ_p the polymer relaxation time, ν_p the polymer zero-shear-stress viscosity and $\boldsymbol{f}$ a forcing term. The incompressibility condition is $\mathrm{div}\ \boldsymbol{u} = 0$. We use a symmetric factorization of the conformation tensor [3], which ensures $\boldsymbol{C}$ to remain positive definite. In order to accurately resolve the dissipation range, we simulate low Reynolds number flow (with a Taylor-scale Reynolds number of order $R_\lambda \approx 30$) using 256^3 grid-points. The relaxation time is varied, but we only present here simulations with $\tau_p = 1s$, corresponding to a Weissenberg number (based on the integral length and velocity scale) of $Wi \approx 1$. The maximum stretching-length $L = 255$ and the ratio of solvent to total viscosity is fixed at 0.909.

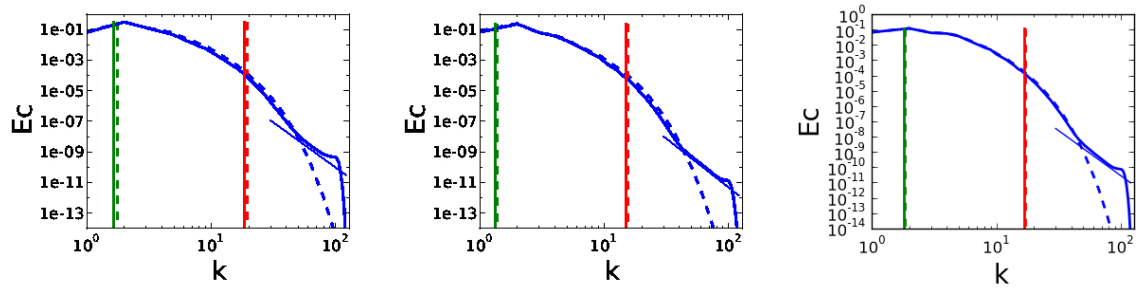
In order to investigate the issues in the following (dissipation range behaviour and drag-reduction), we consider five different flows at comparable Reynolds and Weissenberg numbers. Two of these flows are forced to obtain a statistically steady state, the remaining three are decaying, starting from different initial conditions. We summarize them here:

- Case (a) : stochastically forced [4].
- Case (b) : deterministically forced by a fully helical (ABC) acceleration [5].
- Case (c) : decaying from a steady state of a stochastically forced Newtonian fluid. The polymer is initially at rest i.e $\boldsymbol{C}(t = 0) = \boldsymbol{I}$.
- Case (d) : decaying from a random synthetic field with the same energy distribution as in (c). The polymer is initially at rest i.e $\boldsymbol{C}(t = 0) = \boldsymbol{I}$.
- Case (e) : decaying from a state where both velocity and conformation fields are in a steady state (case (a)); the polymer is thus initially stretched. For the Newtonian case, we remove the polymer stress when the decay begins.

For each case we consider a flow with and a flow without polymers. The comparisons of these different cases allows us to consider the issues discussed below.

INFLUENCE OF VISCOELASTICITY ON SMALL SCALE STATISTICS

A recent investigation has shown that in simulations of the FENE-P model the dissipation range of the kinetic energy distribution is significantly influenced, displaying the presence of a k^{-6} power-law [1]. In order to obtain a better understanding of the turbulence-polymer interaction we further investigate this range in different flow simulations. We show in Figure 1 the energy spectra for cases (a),(b) and (c). Case (a) and (b) are in a statistically steady state and case (c) is evaluated after 1.5 initial turn-over times.



case (a) case (b) case (c)

Figure 1: Wavenumber spectrum of the kinetic energy for cases (a)-(c). The vertical lines indicate the Kolmogorov scale (red) and integral scale (green). Their values are roughly independent of the presence of polymers. The solid lines indicate a k^{-6} slope in case (a) and (c) and $k^{-6.5}$ slope in case (b).

As we can observe, every configuration demonstrates a well pronounced power-law slope close to k^{-6} for wavenumbers beyond the Kolmogorov scale (as also observed in [1] for one particular case). This behaviour is a signature of the back-reaction of the polymers on the velocity field at small scales. During the Colloquium, these observations and its interpretation will be further discussed along with the budget of energy transfer between the two fields. Moreover, we will show that the polymer energy shows also a robust scaling law, proportional to $k^{0.25}$ independent of the flow configuration. This scaling reflects how the polymer energy is distributed over scales.

DRAG REDUCTION

The second issue we discuss is drag-reduction. Whereas its definition is quite intuitive in channel flow, for example, where we can simply measure the change in pressure drop for a given flow rate, its equivalent in isotropic turbulence is conceptually more difficult to understand. One possible definition was introduced in ref. [1], who defined the drag reduction as the difference between the dissipation in a Newtonian ϵ_ν^f flow and the dissipation ϵ_ν^p of the equivalent non Newtonian flow, with the same initial condition, and same parameters:

$$DR = \left(\frac{\epsilon_\nu^f - \epsilon_\nu^p}{\epsilon_\nu^f} \right). \quad (3)$$

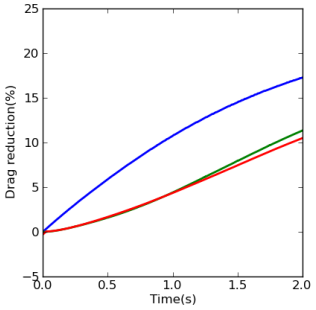


Figure 2: Drag reduction for the simulations (c) (green line), (d) (red line) and (e) (blue line).

The question we want to answer is how this measure of drag-reduction depends on initial conditions in decaying turbulence. For all results the initial turnover-time is unity in dimensionless units and the time-evolution is expressed in such units. In figure 2, we compare the 3 simulations (c,d,e) with different initial conditions. Comparing case (c) and (d) we see that the detailed structure of the velocity field does not significantly affect the drag-reduction. However, the case in which the polymers are initially stretched shows a large difference with the two other cases. This shows that a correct definition of initial conditions for the polymer is crucial to evaluate the drag-reduction in isotropic turbulence using definition (3).

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