TEMPERATURE FLUCTUATIONS INDUCED BY FRICTIONAL HEATING IN ISOTROPIC TURBULENCE

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Abstract The temperature fluctuations generated by viscous dissipation in an isotropic turbulent flow are studied using direct numerical simulation. It is shown that the scaling of their variance with Reynolds number is at odds with predictions from recent investigations. The origin of the discrepancy is traced back to the anomalous scaling of the dissipation rate fluctuations. Phenomenological arguments are presented which explain the observed results.

INTRODUCTION

Turbulence allows for an efficient conversion of kinetic energy into heat. In general, in the presence of external heat-sources, the influence of this heat-production is small, and few studies have considered how these heat fluctuations are exactly redistributed over scales by the turbulent flow itself. The question is however of fundamental scientific importance, and can play a role in practical applications. Recent studies [5, 1], addressed the question of viscous heat generation using spectral models of the turbulence dynamics. These models predict that the variance of the temperature fluctuation will be inversely proportional to the Reynolds number.
The temperature fluctuations are closely related to the dissipation rate fluctuations, a highly intermittent quantity with implications as predicted by Kolmogorov in 1961 (see for example the book of Frisch [6]). Phenomenological arguments taking into account this spottiness lead to the prediction that the temperature fluctuations are correlated at large scales, unlike predictions form closure [3]. We investigate the implication of this by direct numerical simulation.

FRICTIONAL HEATING AND THE DISSIPATION RATE FLUCTUATIONS

We introduce the Reynolds decompostion and write the equation for the fluctuation of the temperature $\theta = \Theta - \bar{\Theta}$,

$$\frac{\partial \theta}{\partial t} + u_i \frac{\partial \theta}{\partial x_i} = \alpha \frac{\partial^2 \theta}{\partial x_i^2} + \frac{1}{c_p} \epsilon', \tag{1}$$

where a bar indicates an ensemble average, c_p is the specific heat, α the thermal diffusivity, and u_i an isotropic turbulent velocity field. The fluctuation of the dissipation rate is indicated by ϵ'. The variance evolves as

$$\frac{d\overline{\theta^2}}{dt} = -\epsilon_\theta + P_\theta \tag{2}$$

with ϵ_θ the dissipation of temperature variance and

$$P_\theta = \frac{2}{c_p} \overline{\epsilon' \theta}. \tag{3}$$

Ignoring the deformation of the scalar blob, the Lagrangian evolution of θ can be formally written, according to (1), as

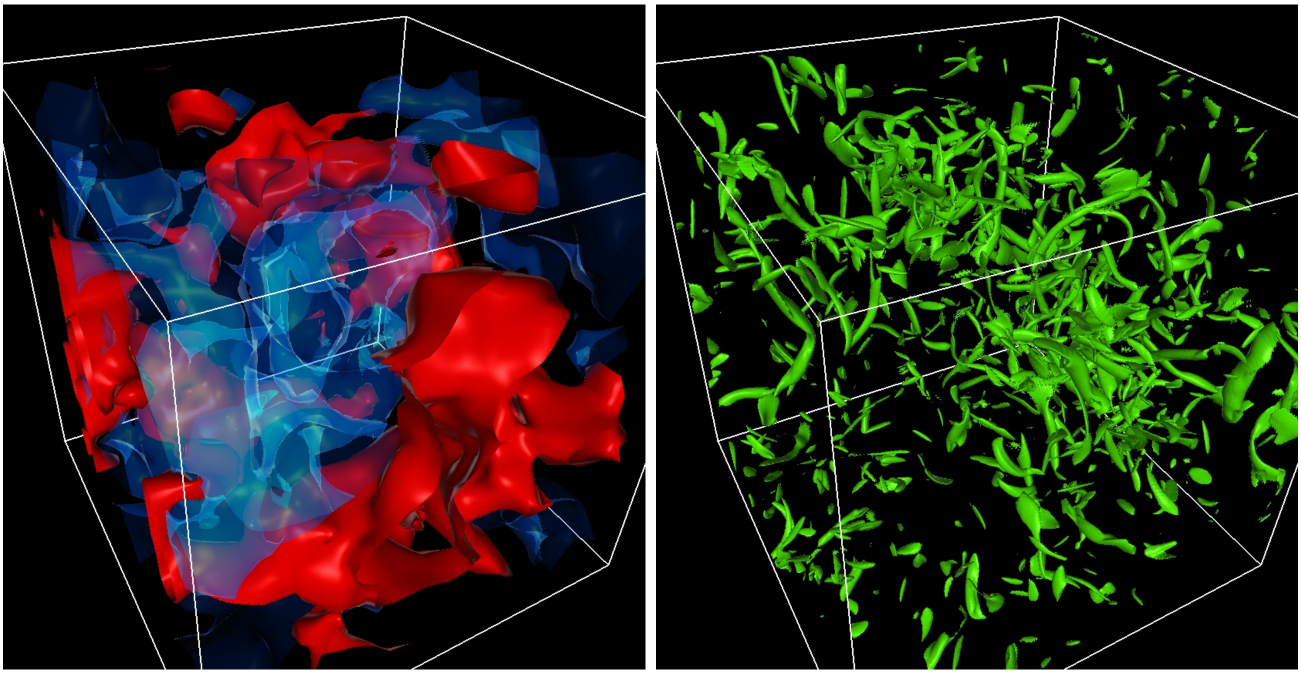
$$\theta(\boldsymbol{x}, t) = \theta(\boldsymbol{x}, 0) + \int_0^t g_\theta(\boldsymbol{x}, t, s) \frac{1}{c_p} \epsilon'(\boldsymbol{x}, s) ds, \tag{4}$$

with $g_\theta(\boldsymbol{x}, t, s)$ the Lagrangian Green's function, so that the production term is given, for long times by

$$P_\theta \approx \frac{2}{c_p^2} \int_0^t \overline{g_\theta(\boldsymbol{x}, t, s) \epsilon'(\boldsymbol{x}, s) \epsilon'(\boldsymbol{x}, t)} ds. \tag{5}$$

Assuming the time-correlations in this expression to be of exponential form, and with a correlation time T, we find that,

$$P_\theta \sim \frac{T}{c_p^2} \overline{\epsilon'^2}. \tag{6}$$



(a) (b)

Figure 1. Visualizations, isosurfaces of (a) the temperature field (red: hot; blue: cold) (b) the vorticity field (iso-surfaces of the enstrophy). Visualizations by VAPOR [4].

This relation shows that the production of temperature variance is proportional to the variance of the dissipation rate fluctuations. This latter quantity is not well predicted by approaches of the Direct Interaction Approximation type, so that we can expect that the same holds for the temperature fluctuations. In particular, $\overline{\epsilon'^2}$ is shown in experiments and simulations to be a quantity correlated at large scales (e.g. [7]), and invariant with Reynolds number at high values of the latter. We expect therefore that, in contrast to the results in [1], the variance of the temperature fluctuations is given by

$$\overline{\theta^2} \sim \frac{U^4}{c_p^2}. \tag{7}$$

With U the rms velocity. The details of the derivation are explained in the full paper [2], where it is shown that this is indeed observed. In Fig 1 results from Direct Numerical Simulations illustrate qualitatively that that the temperature fluctuations are indeed correlated at large scales. Indeed, comparing the temperature fluctuation isosurfaces with those of the enstrophy, it is observed that the former are smooth, whereas the latter are highly spotty and correlated at a smaller lengthscale. Quantitative measures consolidating these results are presented in the full paper [2]. In particular it is shown that the temperature variance becomes independent of the Reynolds number, in disagreement with the previous predictions. The implication is that the temperature variance in very high Reynolds number turbulence can be several orders of magnitude larger than previously predicted.

References

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