# LARGE-EDDY SIMULATION OF A SEPARATED FLOW WITH A SUB-FILTER SCALE MODEL BASED ON THE INTEGRAL LENGTH-SCALE

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<u>Abstract</u> A new sub-filter scale model for large-eddy simulations, which uses a length-scale proportional to the integral scale of the turbulence instead of the grid resolution to parametrize the modelled stresses, will be assessed in the prediction of the flow of a boundary-layer over a rough surface, which includes separation and reattachment.

#### **INTRODUCTION**

Large-eddy simulation (LES) has been a valuable tool in the numerical study of fluid flows. One of its advantages is that it uses the grid resolution to solve the large turbulent scales, which depend on the flow geometry, while Reynolds averaged Navier-Stokes (RaNS) require modelling of all the turbulent scales. As such, most subgrid-scale (SGS) models depend explicitly on the grid resolution as, for instance, the Smagorinsky model:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2\nu_{\rm t}\overline{S}_{ij} = -2(C_S\Delta)^2 |\overline{S}|\overline{S}_{ij} , \qquad (1)$$

where  $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$  characterizes the grid resolution. However, occasionally the link between the grid resolution and the subgrid-scale stresses can be a drawback. For instance, if a wall-model is used, the small-scales should be "sufficiently" filtered near the surface [4], which can prevent the utilization of high-resolution grids. Or, if the grid resolution varies quickly, as possible with unstructured or multi-block grids, the eddy-viscosity will have sharp gradients and can cause numerical instabilities or increase excessively the commutation error (between filtering and derivation). Studies of discretization and modelling errors require also additional care, because refining the grid, to decrease the discretization error, will allow for a larger range of scales to be resolved and decrease the SGS stresses while varying the magnitude of the modelled stresses can mean changing the grid resolution.

In an attempt to make sub-filter stresses independent from the grid resolution, recently Piomelli *et al.*[5] proposed a sub-filter scale model where the length scale is related to an integral length-scale approximation (ILSA) of the turbulence, instead of the grid resolution:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2\left(C_k \frac{k^{3/2}}{\varepsilon}\right)^2 |\overline{S}|\overline{S}_{ij}.$$
(2)

The model was tested using isotropic turbulence and a plane channel flow. The value of the parameter  $C_k$  was determined prescribing the fraction of the energy dissipation to be resolved, using a set of coarse grid simulations. Its accuracy was similar to the dynamic Smagorinsky model in a fine grid, but better if the grid was coarse, with a smaller computational cost.

The idea of decoupling the parametrization of sub-filter stresses from the grid resolution is also appealing for the largeeddy simulation of atmospheric flows, as recognized by Mason and Thomson [3]. One of the advantages is that it should allow to avoid the special treatment that some SGS models require near the surface [9, 1, 6], where the modelled stresses may be larger than the resolved ones, which is closer to RaNS than to LES. However, with atmospheric flows, all the turbulent kinetic energy dissipation should be accounted for the sub-filter scale model and it is not possible to find an appropriate value for  $C_k$  in the ILSA model prescribing the fraction of the dissipation to be resolved. An alternative can be specifying a fraction of the Reynolds stresses that should be resolved. Considering that Smagorinsky-like models account only for the anisotropic part of the tensor, a parameter  $s_{\tau}$  can be defined [5]:

$$s_{\tau} = \left[\frac{\langle \tau_{ij}^{a} \tau_{ij}^{a} \rangle}{\langle (R_{lm}^{a} + \tau_{lm}^{a})(R_{lm}^{a} + \tau_{lm}^{a}) \rangle}\right]^{1/2} , \qquad (3)$$

measuring the resolved fraction of the anisotropic part of the Reynolds stress tensor. Prescribing  $s_{\tau}$  it is possible to determine an appropriate value for the coefficient  $C_k$ .  $s_{\tau}$  can be either constant for the whole computational domain or, for instance, increase close to the surfaces.

The objective of the work to be presented is to verify the accuracy of the ILSA sub-filter model in the large eddy simulation of the boundary layer flow over a smoothly contoured ramp, which is studied experimentally by Song and Eaton [8]. The prediction of such flow is especially challenging since it occurs over a rough surface, requiring the utilization of a wall-model as in the applications of the atmospheric boundary layer, and the separation is controlled by the adverse pressure gradient that develops along the ramp. The results will also be compared with the dynamic Smagorinsky model.

### MATHEMATICAL MODEL AND NUMERICAL TECHNIQUES

The filtered continuity and Navier-Stokes equations are discretized in a non-staggered grid using the finite volume approach. Second order central differences are used in the approximation of both convective and diffusive fluxes and the time discretization uses a fractional step procedure, where the momentum equations advance in time with an explicit, third-order accurate, Runge-Kutta scheme. The code has been used in other studies of atmospheric flows with the Lagrangian dynamic SGS model [6, 7], which here will be supplemented by the ILSA sub-filter model previously described.

## RESULTS

Preliminary results were obtained for a fully developed boundary-layer flow over a rough surface ( $z_0 = 3$  cm), using a computational domain with dimensions 6000 m × 3000 m × 1000 m, a canonical atmospheric flow. Three different grid resolutions were used, with 64, 96 or 128 nodes along each direction, and periodic boundary conditions were applied in the streamwise and spanwise directions.

Comparing velocity profiles obtained with the ILSA and the Lagrangian dynamic scale-dependent models (figure 1a), it was found that both models agree and the log-law is correctly predicted if  $s_{\tau} \approx 2 \times 10^{-2}$  was used with the ILSA model, while increasing  $s_{\tau}$  leads to a higher intercept of the log-law. This aspect is similar to hybrid RaNS/LES models: if the turbulent fluctuations are damped excessively near the surface (higher eddy-viscosity), the velocity gradient increases to increase the turbulent production, originating a "buffer-layer" [2]. With the ILSA model, prescribing the same  $s_{\tau}$  in all the domain resulted in a  $C_k$  nearly constant (figure 1b), meaning that the sub-filter stresses were characterized by a length-scale that was a constant fraction of the integral, while with the scale-dependent Lagrangian dynamic model the subgrid stresses account for an increased fraction of the integral scale when the surface is approached.



Figure 1. (a) Mean velocity profiles (with different grids) and (b)  $C_k$  coefficient (only  $128 \times 128 \times 128$  grid) obtained with the ILSA sub-filter scale model and the scale-dependent Lagrangian dynamic subgrid scale model.

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