BOUNDARY-LAYER-FLOW INSTABILITY IN A RAPIDLY ROTATING AND STRONG PRECESSING SPHERE

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Abstract The linear stability analysis of the steady flow is performed in a rapidly rotating sphere with strong precession. It is shown that the localized mode destabilizing the boundary-layer flow determines the stability boundary, giving the asymptote, $Po \propto Re^{2/3}$, which is consistent with the results obtained by direct numerical simulation.

A precessing sphere We consider the flow of an incompressible viscous fluid in a sphere which is spinning with a constant angular velocity Ω_s and precessing with another constant angular velocity Ω_p perpendicular to the spin (see Figure 1). The flow properties of this system are characterized by two nondimensional parameters, the Reynolds number $Re = a^2\Omega_s/\nu$ and the Poincaré number $Po = \Omega_p/\Omega_s$, where a is the sphere radius and ν is the kinematic viscosity of fluid. Although this canonical flow has long attracted peoples's attention as a simple model of rotating celestial bodies especially with relation to the geophysical applications as well as the compact turbulence generator, the fundamental properties, such as the structure of the steady flows and their instability boundaries over the whole parameter range have not been studied systematically yet. Here we investigate the stability characteristics of the steady flow of this system.

Stability of steady flows Since the steady flow in a precessing sphere with arbitrary values of Re and Po is not simple enough to be expressed analytically, the stability analysis must be performed numerically. We are currently performing the stability analysis by direct numerical simulation, the stability boundary obtained so far is shown in Figure 2 with dots, the right (or left) side of which is stable (or unstable). The current status of computer power inevitably limits us the calculation for finite values of Po roughly in the range $0.06 < Po < 1.4$. The asymptotic analysis would be useful beyond this region. For lower region ($Po \ll 1$) we obtained the asymptote ($Po = 21.25Re^{-4/5}$) which agrees excellently well with experiment (not shown in Figure 2 but in Figure 1 of [2]). Here we consider the upper region ($Po \gg 1$), namely the strong precession limit.

Flow structure in the strong precession limit It is well-known [1] that the flow is almost still in the precession frame of reference in the limit of strong precession except for the thin boundary layer. Tthe thickness δ of the boundary layer is of $O((RePo)^{-1/2})$ except for the critical region around the great circle perpendicular to the precession axis (see the gray circular belt in Figure 1) where the boundary-layer approximation breaks down. The thickness and width of this critical region are of $O(\delta^{4/5})$ and $O(\delta^{2/5})$, respectively. In terms of the spherical polar coordinates (r, θ, ϕ) (with θ being the polar angle from the z axis and ϕ being the azimuthal angle from $-y$ axis) we introduce stretched coordinates (ξ, η) by $r = 1 - \delta^{4/5}\xi$ and $\cos\theta = \delta^{2/5}\eta$. The radial the polar and the azimuthal components of velocity, (u_ϕ, u_ξ, u_η), are of $O(\delta^{2/5})$, $O(\delta^0)$ and $O(\delta^0)$, respectively. All of these three components are proportional to $\cos\phi$, and their distributions on the $\phi = 0$ plane are plotted in Figures 3 and 4.

Origin of the instability It is important to note here that the inertial waves which may be excited in the still region is neutrally stable in the inviscid limit and that they always decay if the viscous effects (from the boundary-layer flow) are taken into account. Moreover, the precession effects are too small to destabilize the inertial waves as long as the nonlinear interactions are neglected. Thus we are tempted to examine the instability of the flow in the critical region of the boundary layer.

Linear stability of the critical-region flow The disturbance equations can be derived easily by taking the linear terms in the Navier-Stokes equations and the continuity equation. By taking account of the above-mentioned scalings in length and velocity in the critical region and picking up the leading orders of the nonlinear term (having destabilizing effects) and the viscous term (having stabilizing effects) we find $Po = \gamma Re^{2/3}$, where γ is an unknown constant to be determined by solving the eigenvalue problem of the disturbance. This calculation is under way, and the value of γ will be presented at the conference. For reference we draw the power law $Po \propto Re^{2/3}$ in Figure 1.

References

[1] Greenspan, H.P. The theory of rotating fluid.*Cambridge University Press* 1968.
[2] Kida. S. Localized unstable modes in a precessing sphere. *ETC14* Abstract **164** (2013).

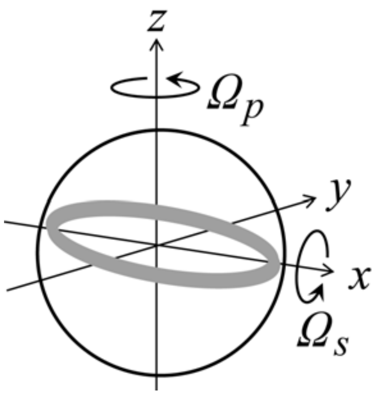

Figure 1. A precessing sphere spinning with angular velocity Ω_s around the x-axis which is rotating with angular velocity Ω_p around the z-axis. The gray circular belt represents the critical region of the boundary layer.

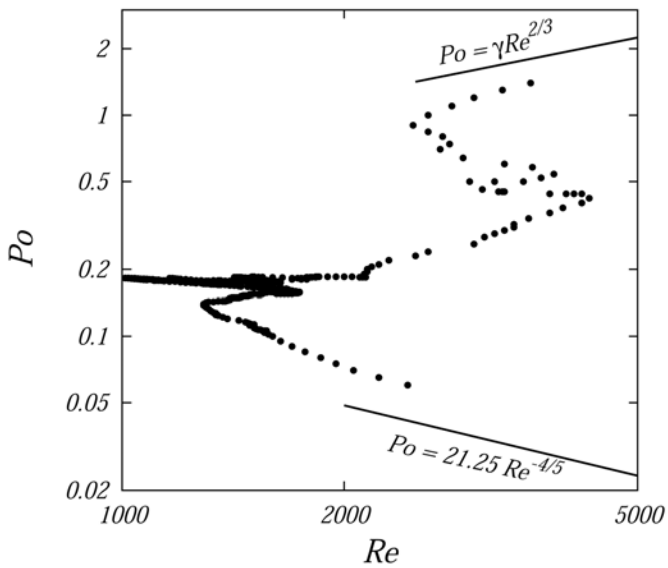

Figure 2. Stability boundary of the steady flow. The flow is stable or unstable in the left or right side of the line of dots which were obtained by direct numerical simulation. The lower asymptote $Po = 21.25Re^{-4/5}$ is taken from [2], whereas the upper one $Po = \gamma Re^{2/3}$, where γ is an unknown constant to be determined, is the present result.

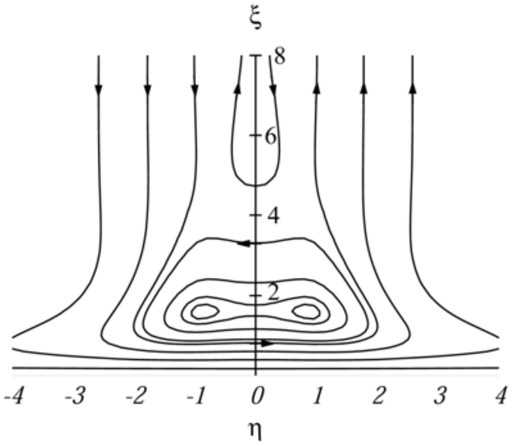

Figure 3. The streamlines of (u_η, u_ξ) on the $\phi = 0$ plane. The arrows indicate the flow direction.

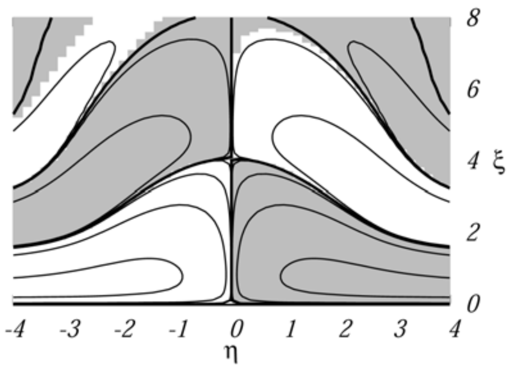

Figure 4. The contours of u_ϕ on the $\phi = 0$ plane. The white and gray areas indicate $u_\phi > 0$ and $u_\phi < 0$ respectively.