TURBULENT BURSTS AND LINEAR INSTABILITIES IN ROTATING CHANNEL FLOW

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Abstract Recurring bursts of turbulence are observed in DNS of rotating channel flow for a range of Reynolds numbers and rotation rates. The bursts are caused by a linear instability and happen in a weakly to strongly turbulent environment. In some cases turbulence or other flow fluctuations can slow down the instability.

DNSs of fully developed turbulent rotating about the spanwise axis have been carried for $Re = U_b h/\nu = 3000$ up to 31600 and $Ro = \Omega h/U_b = 0$ up to about 3. Here U_b is the bulk mean velocity, h the channel half gap width and Ω the rotation rate. When Ro is raised turbulence becomes progressively weaker on one side of the channel but is still intense on the other, leading to asymmetric velocity and Reynolds stress profiles. However, when Ro nears 2.7 to 3 turbulence becomes weak in the whole channel, in agreement with [1, 2], and the velocity approaches a laminar Poiseuille profile.

A striking phenomenon is that for a range of Re and Ro strong, reoccurring bursts of turbulence are observed in the DNSs of rotating channel flow. Evidence for these cyclic bursts are given by e.g. distinct peaks in the time series of the turbulent kinetic energy K (figure 1). The time scale of the turbulent bursts is $O(1000\,t)$, where $t = h/U_b$, which is much longer than the turbulence time scale of $O(1\,t)$ and rules out a direct role of turbulence.

Visualizations show that before a turbulent burst a large-scale plane wave appears in the flow (figure 2). In some DNSs like the one visualized in figure 2 the wave is mostly confined to the side with the weakest turbulence while the other channel side is strongly turbulent, but when Ro approaches 2.7 to 3 and turbulence is weak everywhere the waves are clearly noticeable in the whole channel. The amplitude of the wave grows approximately exponentially and after some time when the wave has a large amplitude a secondary instability sets in and the wave completely breaks down causing a turbulent burst [3]. Due to the rotation the turbulence rapidly decays and after some time when the turbulence is weak the plane wave re-emerges. The whole process repeats itself leading to a continuous cycle of turbulent bursts [3]. This cycle of bursts is not triggered by some external perturbation; only the imposed mean pressure gradient and rotation are needed to drive the cyclic turbulent bursts.

The plane wave instability seen in the visualizations bears similarities with a linear Tollmien-Schlichting (TS) wave instability seen in laminar flows. A linear instability analysis shows that waves with streamwise wavenumber $\alpha \sim 1$ (normalized with h) and spanwise wave number $\beta = 0$ like the one seen in the visualizations are indeed linearly unstable when cyclic bursts are observed. In the analysis the base flow is taken equal to the mean flow of the DNS. The exponential growth rate and eigenfunction of the plane wave in some DNSs closely matches the linear stability analysis [3], see figure 3.*a*. However, in other DNSs the exponential growth rate of the plane wave causing the bursts is lower than in the linear instability analysis (figure 3.*b*) and in some DNSs no growing plane waves and turbulent bursts are observed although the mean flow is linearly unstable.

The mismatch between some observations and linear stability analysis indicates that in some DNSs of rotating channel flow turbulence or other flow fluctuations have an influence on the linear instability. The evolution equation for the kinetic energy of a plane wave in Fourier-space shows that the growth of a TS wave in laminar Poiseuille flow is basically governed by a production term $\mathcal{R}e(-\hat{u}^*\hat{v}\ \partial U/\partial y)$ (energy is extracted from the mean flow U) and a dissipation term because there is no energy transfer to other modes by pressure and triad interactions. Here, $\hat{u}$ and $\hat{v}$ are the streamwise and wall-normal Fourier modes of the TS wave and $\mathcal{R}e$ denotes the real part. Examination of the DNS data makes clear that in none of the DNSs there is a significant energy transfer from the plane wave to other modes by nonlinear triad interactions before the secondary instability sets in, i.e. in all cases with cyclic turbulent bursts the growth of the plane wave is determined by a balance between the production and dissipation term. In this respect, the observed instability can be considered linear in all cases. Instead the mismatch between observation and linear stability analysis in some DNSs is caused by a difference between $\hat{u}^*\hat{v}$ observed in the DNS and $\hat{u}^*\hat{v}$ predicted by linear stability analysis. This indicates that in some DNSs turbulence or other flow fluctuations modify the plane wave, i.e. $\hat{u}^*\hat{v}$. As a consequence, the kinetic energy production term and thus the growth rate of the plane wave is reduced.

In short, recurring bursts of turbulence are observed in DNSs of rotating channel flow for a range of Re and Ro. The bursts are caused by a linear instability producing rapidly growing plane waves. In some cases, when Ro is not too high, the linear instability happens in a clearly turbulent environment. In none of the DNSs with cyclic bursts a significant energy transfer from the rapidly growing plane wave to other modes by nonlinear triad interactions is observed as long as the secondary instability has not set in yet. However, turbulent and other flow fluctuations can modify the plane wave and thereby slow down its growth rate. The present case can have a broader relevance since it may give insights into linear instabilities occurring in turbulent environments.

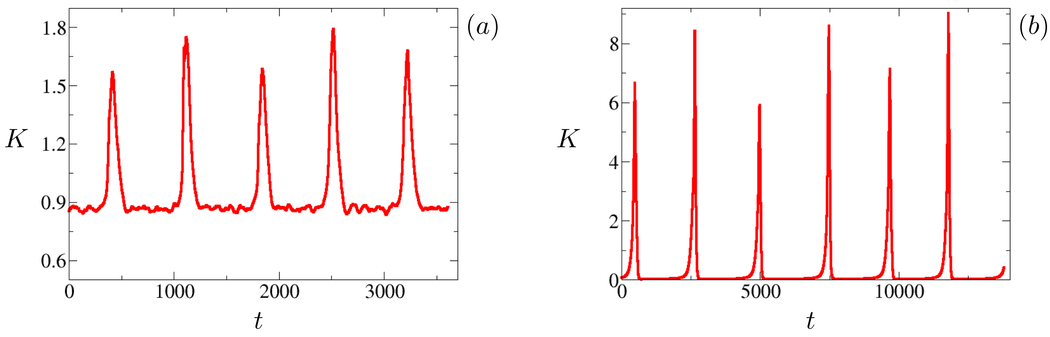

Figure 1. Time series of the turbulent kinetic energy K integrated over the whole channel domain at (a) $Re = 31600$ and $Ro = 1.2$ and (b) $Re = 20000$ and $Ro = 2.4$. Time t is normalized by h/U_b.

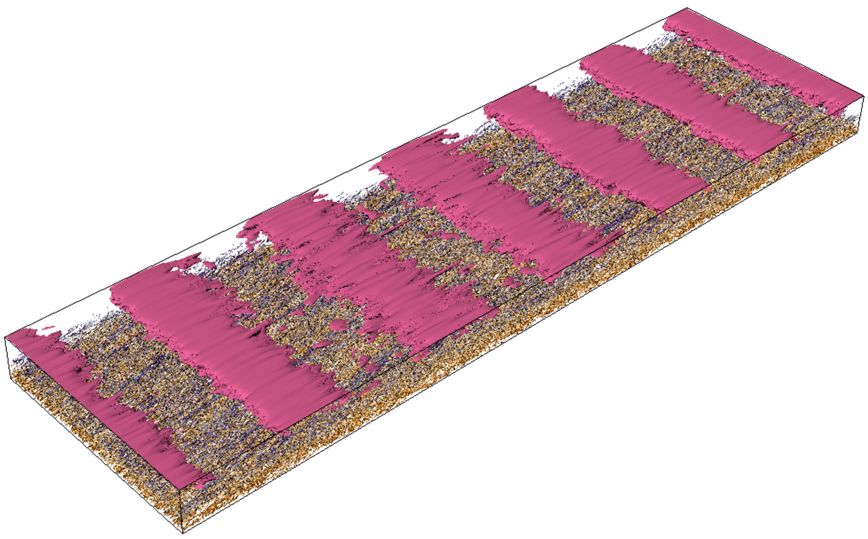

Figure 2. Three-dimensional visualization of the plane wave and turbulent vortices (top and bottom of the channel respectively) in a DNS of rotating channel flow at $Re = 31600$ and $Ro = 1.2$. The flow is from the bottom-left to the top-right corner.

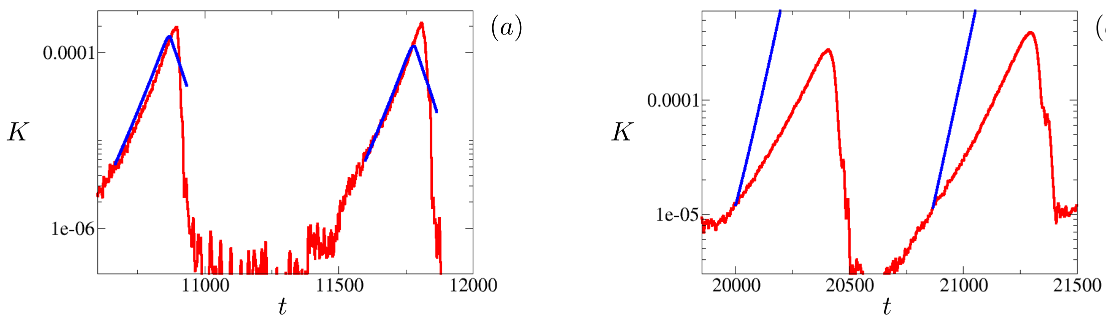

Figure 3. Energy of the plane wave causing the turbulent bursts in the DNS (red line) and predicted by a linear stability analysis (blue lines) at (a) $Re = 20000$ and $Ro = 1.2$ and (b) $Re = 10000$ and $Ro = 1.8$.

References

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[3] G. Brethouwer, P. Schlatter, Y. Duguet, D.H. Henningson, and A.V. Johansson. Recurrent bursts via linear processes in turbulent environments. *Phys. Rev. Lett.* **112**: 144502, 2014.