TURBULENT BANDS IN A PLANAR SHEAR FLOW WITHOUT WALLS

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Abstract The banded structure of turbulence is observed immediately beyond transition in shear flows with two unconstrained directions (e.g. TCF, PCF, PPF). Yet despite its ubiquitous nature, the mechanisms underpinning bands are not understood to the level of localized turbulence in pipe flow. To this aim we investigate turbulent bands in Waleffe flow, a sinusoidal shear flow, $U(y) = \sin\left(\frac{\pi}{2}y\right)$, with stress-free boundary conditions at $y = \pm 1$. The existence of turbulent bands in this system demonstrates that walls are not necessary to induce the phenomenon. The sinusoidal nature of the base forcing means the dominant features of bands can be viewed through a small number of Fourier modes in y. Utilizing this simple dependence we examine the emergence of turbulent bands from uniform turbulence.

TURBULENT BANDS

Regular patterns of turbulent and laminar flow are a robust feature found at transitional Reynolds numbers. Studied first in Taylor-Couette flow [2, 4] (but also in plane Couette flow [5] and plane Poiseuille flow [6]) these patterns are not aligned with the shear direction but tilted into the spanwise direction. Bands have been computationally studied in minimal domains which are tilted with an angle θ against the streamwise direction to align with the pattern and thereby require a computational domain with only a single large dimension, denoted z, which is aligned with the wavevector of the pattern [7, 8]. Averaging over time and the band-aligned direction, which we denote by x, reveals coherent structures of rolls and streaks which are maintained by time-dependent Reynolds stresses. It has been proposed that the bands emerge in a linear instability from the time averaged uniform turbulence [7]. Attempts to model this behaviour have thus far been limited and not found the success of modelling localized turbulence in pipe flow [1]. There still remains much work to understand the mechanisms involved in maintaining these patterns which motivates this work.

WALEFFE FLOW

To understand the mechanisms involved in turbulence in small domains, Waleffe [10] studied the sinusoidal shear flow created by body forcing

$$F(y) = \frac{\pi^2}{4Re}\sin\left(\frac{\pi}{2}y\right),$$

with periodic boundary conditions in (x, z) and stress-free conditions at $y = \pm 1$, where y is the "wall"-normal direction. The Reynolds number is defined using the same non-dimensionalization as plane Couette flow. This forcing invokes a laminar flow

$$U(y) = \sin\left(\frac{\pi}{2}y\right),$$

which is linearly stable [9]. This flow is clearly related to Kolmogorov flow, however the imposition of stress-free boundary conditions removes the instabilities associated with that flow.

RESULTS

To study bands in this flow we use `ChannelFlow` [3] to simulate a domain tilted at $\theta = 24^\circ$ and of size $L_x \times L_y \times L_z = 10 \times 2 \times 40$, which matches those used in studies of Couette and Poiseuille flow [7, 8]. We find turbulent bands over a wide range of Reynolds numbers $Re \in [125, 350]$. These bands qualitatively match those found in plane Couette flow (figure 1) but differ in the near-wall regions due to the differing boundary conditions. Plotting flow profiles $U(y)$ at varying z (figure 2), we see that Waleffe flow captures the behaviour of PCF away from the walls. In Waleffe flow, the sinusoidal laminar flow and stress-free boundary conditions induce dynamics which can be captured with a small number of Fourier modes in the "wall"-normal direction. Utilizing this and motivated by Waleffe's self-sustaining process [10] we attempt to model the development of bands from the time-averaged uniform turbulent state found at higher Reynolds numbers.

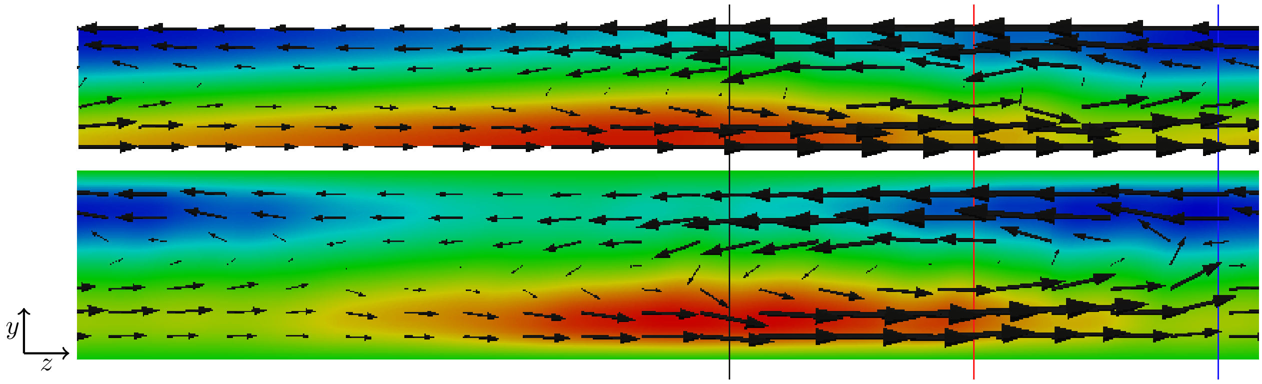

Figure 1. Turbulent bands in Waleffe flow (top) and plane Couette flow (bottom) in the (z, y) plane, arrows denote within-plane flow while colour denotes flow into and out of the plane. Direction y has been scaled up by factors of 2 and 3.2 respectively, in Waleffe for ease of view and in PCF motivated by figure 2. Time and x-averaged flow in the tilted domain at *Re* $= 225$ (Waleffe) and *Re* $= 350$ (PCF) shows the striking similarities between the two flows with the flow differing qualitatively only near the boundaries at $y = \pm 1$.

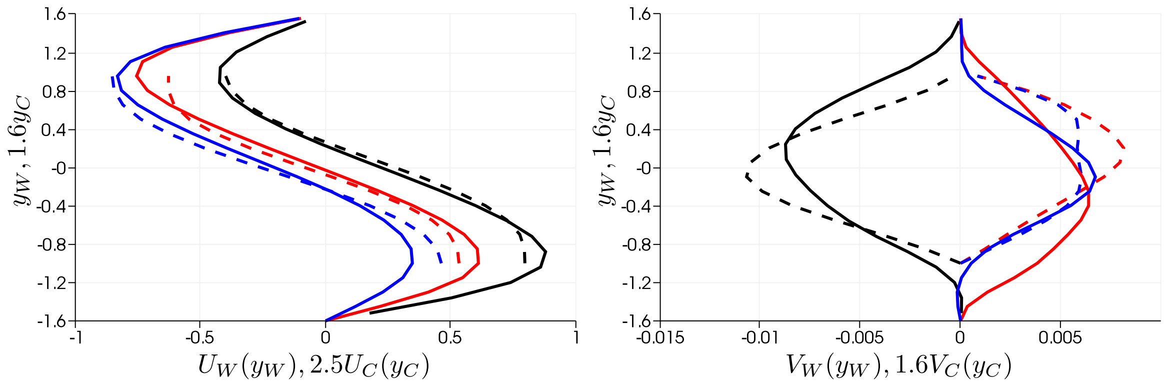

Figure 2. Slices $U(y)$ (left) and $V(y)$ right comparing Waleffe flow (dashed) and PCF (full lines) which correspond to the coloured lines in figure 1. For comparison purposes the y direction has been scaled by 1.6 (and thus plot $1.6V_C(y_C)$). The absence of walls in Waleffe flow has minimal effect upon the band structure, which matches the interior of the bands found in PCF.

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