STOCHASTIC ANALYSIS OF THE EFFECTS OF INLET VELOCITY CONDITIONS ON THE EVOLUTION OF SPATIALLY EVOLVING MIXING LAYERS

Marcello Meldi [1], Maria Vittoria Salvetti[2] & Pierre Sagaut[3]
[1] *PPrime, University of Poitiers, France*
[2] *DIA, University of Pisa, Italy*
[3] *M2P2, University of Aix-Marseille, France*

Abstract The sensitivity of the evolution of the spatially evolving mixing layer test case to inlet conditions parameters is studied in the present work. The analysis is performed combining DNS simulations and a response surface stochastic approach. The angle between the two asymptotic streams θ and the parameter $\alpha = (U1 - U2)/(U1 + U2)$ are identified as parameters of interest and they are considered as random variables. The uncertainty propagated over the output quantities, such as the average velocity U, is then investigated.

The analysis of the results indicate that different physical regimes can evolve from the initial conditions investigated. In particular, non linear effects streaming from interactions between θ and α are observed. These effects are not captured by classical DNS analyses of the spatially evolving mixing layer test case.

INTRODUCTION

Mixing layers are present in many technological applications or environmental flows and they have been extensively studied in the literature. Mixing layers are characterized by the evolution and the interaction of coherent vortex structures, which form at the interface between two parallel coflowing streams with different velocities and play a major role in the bulk mixing of the fluids and in the growth of the mixing layer. It is well known that the inlet conditions, which may be not exactly known in practical applications, significantly affect the dynamics of mixing layers. The most common assumption in studying this flow configuration is that the two coflowing streams are parallel, i.e. the two inlet bulk flow velocities have the same direction; in this case, the coherent vortex structures form due to the effect of the gradient of velocity occurring in the normal direction.

However, it may happen in practice that this condition is not verified and that the two streams are actually tilted of an angle θ. Previous experimental studies [1] have shown that the flow evolution is extremely sensitive to the three-dimensionality of the inlet velocity field. A second parameter influencing the flow transition from the laminar regime to the fully developed turbulent one, as well as the dynamics of the generated eddies [2], is $\alpha = (U1 - U2)/(U1 + U2)$, where $U1$ and $U2$ are the two asymptotic stream velocities.

STOCHASTIC ANALYSIS

In the present work, we propose to quantify the impact on flow evolution of the angle between the two streams, θ and of α through direct numerical simulations and by adopting a stochastic approach, in which the parameters are considered as random variables and the uncertainty is propagated over the output quantities of interest. To reduce the required computational resources, a continuous representation of the investigated flow quantities over the random variable space (response surface) is obtained through a simple, inexpensive model.

The response surface is reconstructed starting from a limited number of samples taken at known discrete points on the uncertainty space, i.e., single deterministic numerical simulations. In particular, as already done for classical mixing-layers in [3], the response surface is built through generalized Polynomial Chaos (gPC).

The deterministic simulations needed to obtain the response surface are direct numerical simulations carried out through the open-source code OpenFOAM. The Reynolds number $Re = \frac{(U1-U2)\delta_0}{2\nu}$ is set equal to 100, in which δ_0 is the inlet vorticity thickness and ν is the kinematic viscosity. The following ranges of variation of the random variables have been chosen: $0 \leq \theta \leq \pi/12$ and $0.2 \leq \alpha \leq 0.7$. As previously mentioned, in our approach, each deterministic simulation corresponds to a discrete point in the two dimensional uncertainty space. These points are the quadrature nodes of a two dimensional Gauss-Legendre quadrature of the uncertainty space. This quadrature rule has been chosen as it accurately fits with the enforced probability density function of the random variables, which has been chosen as uniform. The gPC expansion for the two random variables is truncated at the third order, which means that the minimum number of simulations required for a full convergence of the response surface model is 4^2. The flow quantities of interest, for which the uncertainty propagation is quantified through the response surface, are the three components of the mean velocity vector, U and of its quadratic function $M = (U1 - U)(U - U2)$, which is related to the momentum thickness. The spatial evolution of the two quantities is analyzed for each component as well as for the vector magnitude. The results are as well compared with the information which could be obtained by deterministic DNS simulations, carried out for different values of the input parameters.

The analysis of the results indicate that different physical regimes can evolve from the initial conditions investigated. In particular, non linear effects streaming from interactions between θ and α are observed. These effects are not captured by classical DNS analyses of the spatially evolving mixing layer test case. The output quantity SS_X, which is a measure of the rate of convergence towards a self-similar behavior of the streamwise component of the average velocity U_X, is presented in figure 1. Squares and circles symbols are associated to classical DNS simulations ($\theta = 0$) and $\alpha = 0.2\,,\, 0.7$ respectively. The two dimensional normalized pdf indicates that the most probable evolution of the parameter SS_X does not follow either of the two classical cases going downstream. A significant variance of the results is obtained, which comes from non-negligible interactions between the parameters investigated. Moreover, a bifurcation is observed far from the inlet conditions at high x/Λ values, where Λ is the characteristic length scale of the flow.

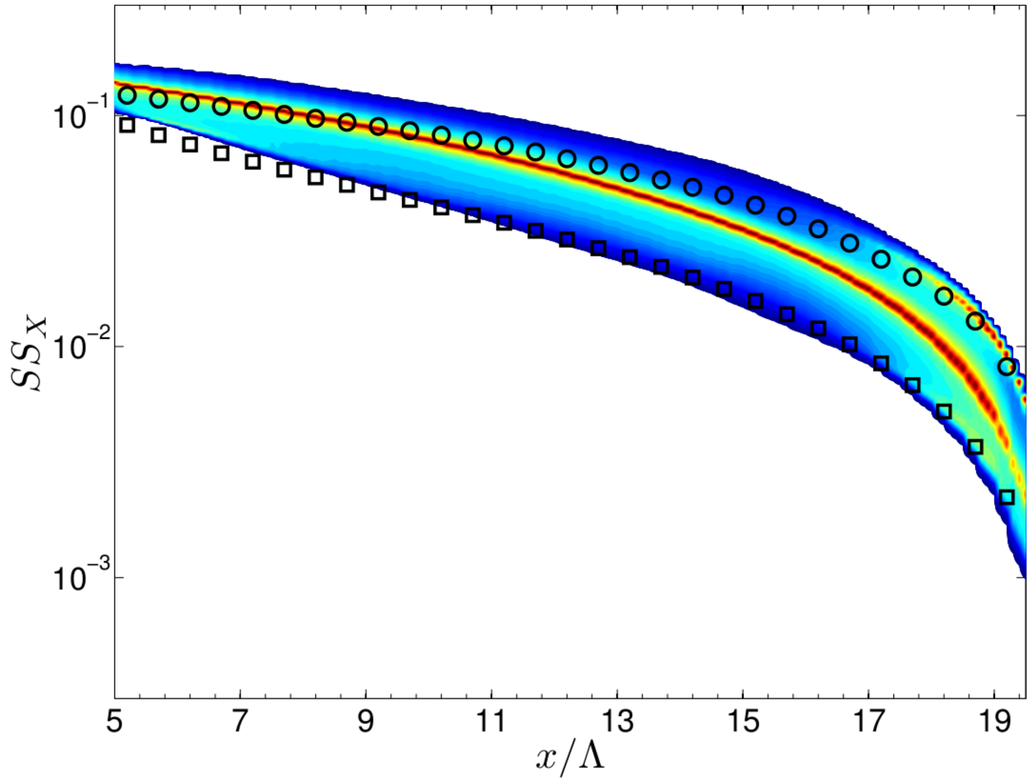

Figure 1. Normalized two dimensional pdf of the output parameter SS_X. This parameter is a measure of the rate of convergence towards a self-similar behavior of the streawise component of the average velocity U_X. The data in squares and circles are classical ($\theta = 0$) DNS data of the same test case for $\alpha = 0.2\,,\, 0.7$, respectively. The data are reported against the normalized distance from the inlet in the streamwise direction x/Λ.

References

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