Mean flow generation by Görtler Vortices in a rotating annulus with librating side walls

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Abstract Longitudinal libration of the cylinder side walls of a rotating annulus in the supercritical regime induces a centrifugally unstable Stokes boundary layer which generates Görtler vortices only in a portion of a libration cycle. We show for the first time that these vortices propagate into the fluid bulk and generate an azimuthal mean flow which is retrograde (prograde) over the outer (inner) cylinder side wall. Direct numerical simulations (DNS) are carried out and Reynolds-averaged equations and kinetic energy budget of mean and fluctuating flow are used as diagnostic equations to discuss the generation mechanism and scaling behavior of the azimuthal mean flow in the fluid bulk.

Introduction

As reported in [1], the formation of columnar structures in rotating turbulent flow along the axis of rotation are attributed to the propagation of inertial waves induced by dispersing energy form the eddies. The generation of inertial waves by Görtler vortices has also been reported in [2] in a cylindrical geometry. Inspired by these works, we try to elaborate the generation of an azimuthal mean flow induced by the Görtler vortices, which is not covered in previous studies (cf. [2], [3]). In order to study the generation mechanism, direct numerical simulations of the fluid flow in a rotating annulus with librating cylinder side wall have been carried out. Inner and outer cylinder side wall librate longitudinally according to: $\Omega(t) = \Omega_0(1+\varepsilon\sin\omega t)$, where Ω_0 is the background rotation rate, ε is the libration amplitude and ω is the libration frequency. A centrifugally unstable Stokes boundary layer forms over the inner and outer cylinder side wall and generates Görtler vortices.

Following the discussion presented in [4] about the angular momentum mixing and using Rayleigh discriminant, we show phenomenologically how Görtler vortices cause mixing and redistribution of the bulk angular momentum. Viewing the system from the co-rotating frame of reference, we discuss the generation mechanism, sign selection, and scaling behavior of the azimuthal mean flow using Reynolds-averaged equations and kinetic energy budgets of mean and fluctuating flow.

Results

Figure 1 (upper panel) shows the visualized Rayleigh discriminant using the numerical data and azimuthal component of the vorticity field at different phases (φ=ωt mod 2π) after the onset of instability. At the phases φ=26π/16, φ=27π/16, and φ=28π/16, Görtler vortices represented by centrifugally unstable regions propagate into the centrifugally stable bulk. Görtler vortices mix with the stable flow in the bulk, thus leading to a uniform distribution of angular momentum, while they decay to the next unstable phase of libration. The opposite-signed vortices in the radial component of the vorticity (Figure 1 lower panel) are an indicator of the nonlinear interactions that cause angular momentum mixing. In the following, to understand the relevance of nonlinear interaction we view the system from the co-rotating frame of reference and discuss the generation mechanism of an azimuthal mean flow.

Figure 2 (left) shows the radial profiles of instantaneous but azimuthally averaged azimuthal velocity ($\langle u_\theta \rangle_\theta$) at different phases during one libration cycle for the cases of outer cylinder libration, and intersecting one of the Görtler vortices. Time mean axial-azimuthally averaged azimuthal velocity ($\langle U_\theta \rangle_{z\theta t}$) is also shown. The radial structure of the profiles at different libration phases shows that turbulent flow generation occurs in the retrograde phase of libration cycle (Figure 2 (left)), while it occurs in the prograde phase of libration cycle for the case of inner cylinder libration (not shown). Moreover, the profile of $\langle U_\theta \rangle_{z\theta t}$ reveals a retrograde azimuthal mean flow outside the Stokes boundary layer with a maximum magnitude in the order of the amplitude of librating boundary.

We investigated the Reynolds-averaged equations with respect to two time scales: (i) developing stage of the Görtler vortices (a portion of libration cycle), and (ii) the long time average which is in the order of several libration cycles or viscous time scale. Owing to statistical homogeneity in the axial and azimuthal directions and that the major mean flow is in the azimuthal direction, we obtain two balance equations corresponding to each time scale. For case (i) we obtain the balance equation $\partial\langle \overline{U_\theta} \rangle_{z\theta} / \partial t \approx -\partial\langle \overline{u'_r u'_\theta} \rangle_{z\theta} / \partial r$, where $\langle \overline{U_\theta} \rangle_{z\theta}$ and $\langle \overline{u'_r u'_\theta} \rangle_{z\theta}$ are the ensemble averages of the azimuthal mean flow and the dominating Reynolds stress component. We found that $\langle \overline{u'_r u'_\theta} \rangle_{z\theta} \geq 0$ for inner and outer cylinder libration cases indicating a flux of angular momentum [4] toward the fluid bulk (cf. Figure 2 (right) for outer cylinder libration). If the gradient of $\langle \overline{u'_r u'_\theta} \rangle_{z\theta}$ outside the Stokes boundary layer (thickness δ) is negative (positive), a prograde (retrograde) mean flow is generated (Figure 2 (right)). For case (ii) we obtain a balance between the gradient of the Reynolds stress term $\langle u'_r u'_\theta \rangle_{\theta zt}$ and the viscous term $E \cdot \partial^2 \langle U_\theta \rangle_{\theta zt} / \partial r^2$ (E is Ekman number). Validating this balance equation using numerical data, we found that the azimuthal mean flow is proportional to $\varepsilon^2 -$, $\omega^{-0.5} -$ and $E^{-0.5} -$ scaling law. The kinetic energy budgets of mean and fluctuating flow exhibit the presence of upscale cascade of energy. Additionally, the three dimensional structure of the instantaneous velocity field and corresponding r.m.s. turbulence intencity values revealed an anisotropic turbulent state, which is in accordance with the argumentation put forward in [4].

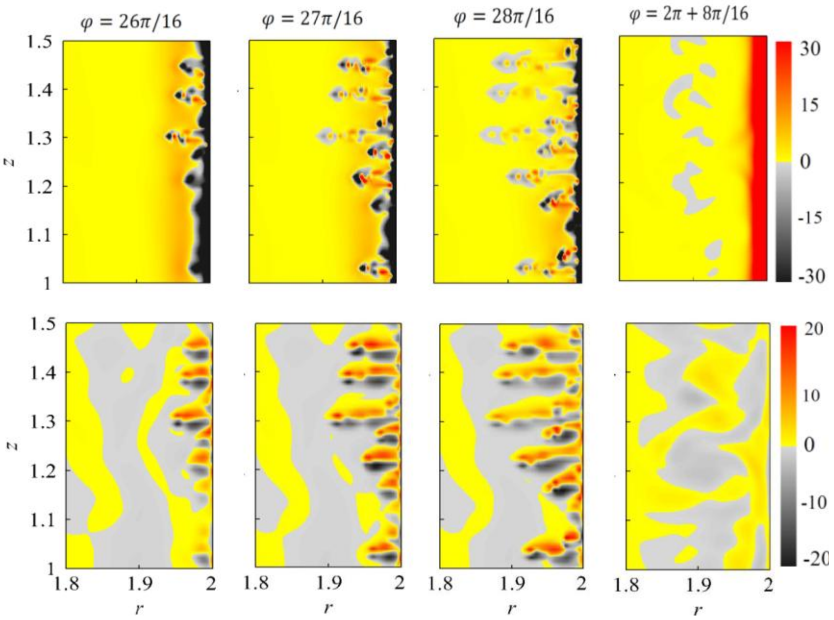

Figure 1. Evolution of the Rayleigh discriminant obtained by numerical data (upper panel) and azimuthal component of the vorticity field (lower panel) at different phases during a libration cycle. The Figure shows the zoomed region at the mid axial height close to the outer cylinder side wall.

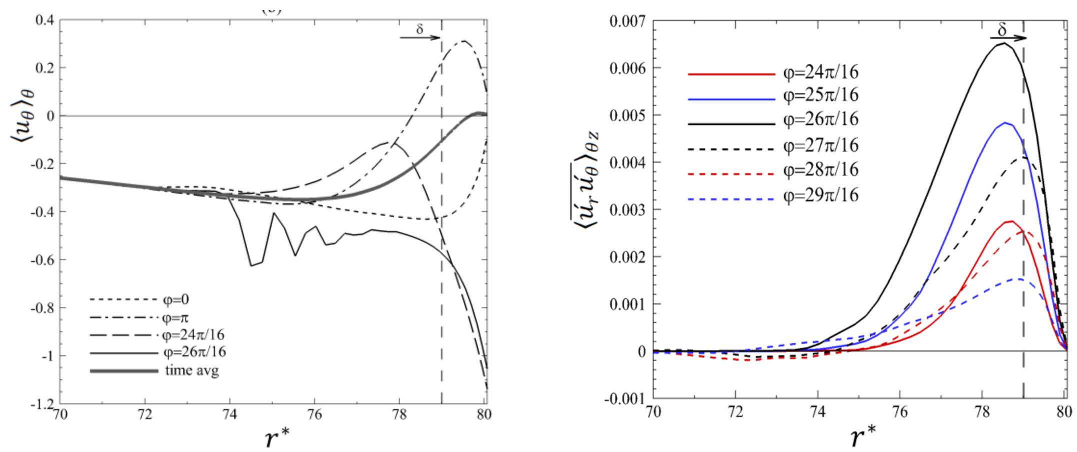

Figure 2. Radial profiles of $\langle u_\theta \rangle_\theta$ at different phases during a libration cycle and $\langle U_\theta \rangle_{\theta zt}$ (thick line) (left panel), and $\left\langle \overline{u'_r u'_\theta} \right\rangle_{z\theta}$ at different phases after the onset of instability for outer cylinder libration (right panel). $E = 4\times10^{-5}$, $\omega = 0.514$, $\varepsilon = 0.6$. r^* is the radius normalized by the thickness of Stokes boundary layer ($\delta \cong 0.0124$).

References

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