DIFFERENTIAL DIFFUSIVE INSTABILITIES OF MISCIBLE TWO-LAYER STRATIFICATIONS IN POROUS MEDIA AND HELE-SHAW CELLS

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Abstract In porous media, a stratification of a given solution on top of another miscible solution can be buoyantly unstable because of an unstable density stratification or because of differential diffusive effects. The former is the well known Rayleigh–Taylor (RT) mechanism wherein the interface is destabilized by the denser solution overlying a less dense one in the gravity field. Whereas the latter is of particular interest in the field of oceanography, when the upper solution is less dense than the lower one with the lower component diffusing faster than the upper one, resulting in a double diffusive (DD) instability. Similarly, a diffusive-layer convection (DLC) instability has also been observed for a stable density stratification with the upper solute diffusing faster than the lower one. Though the literature on differential diffusion effects is pretty vast, very few studies have managed to establish a connection, both qualitatively and quantitatively, between numerical simulations and experimental observations, which is the basis of the present study. We report our findings in a broad parameter range where the instability mechanism could be triggered by an unstable density stratification or due to differential diffusive effects, or even both, resulting in mixed modes [1].

EXPERIMENTAL OBSERVATIONS

Laboratory-scale experiments are typically carried out in vertical Hele-Shaw cells, which consist of two transparent plates separated by a small gap (0.5 mm). For small enough gap widths, the flow evolution in a Hele-Shaw cell is described by Darcy's equations similar to the evolution equation for flows in porous media [2]. We consider a two-dimensional vertical stratification of a solution A at a concentration A_0 and density ρ_A overlying a miscible solution of B at concentration B_0 and density ρ_B with the gravity field pointing downwards in the negative $y-$ direction. The two most important dimensionless parameters are the ratio of the diffusion coefficients, δ, and the buoyancy ratio, R, which are defined as

$$\delta = \frac{D_B}{D_A} \qquad \text{and} \qquad R = \frac{\alpha_B B_0}{\alpha_A A_0}, \tag{1}$$

where D_A and D_B are the diffusion coefficients, α_A and α_B are the solutal expansion coefficients.

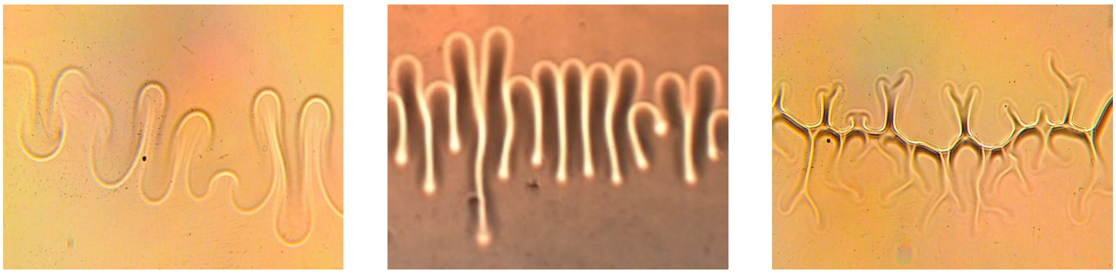
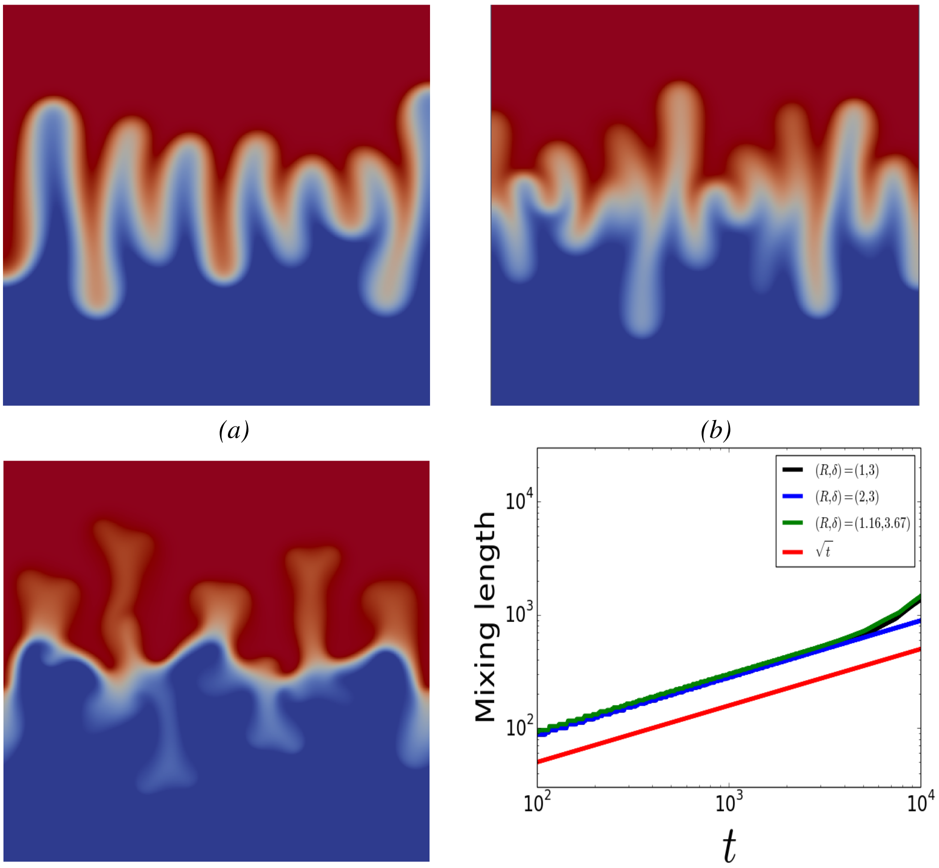

(a) *(b)* *(c)*

Figure 1. Buoyantly driven instabilities obtained in a vertically oriented Hele-Shaw cell: *(a)* Rayleigh–Taylor instability corresponding to $(R, \delta) = (0.75, 0.4)$, *(b)* double-diffusive instability corresponding to $(R, \delta) = (1.16, 3.167)$, *(c)* mixed-mode instability (DLC + RT) corresponding to $(R, \delta) = (0.9, 0.3)$.

The dynamics of the contact line between both solutions is visualized through a Schlieren technique by tracking the changes in the refractive index of the convective motions. Various buoyantly driven instabilities observed experimentally are shown in figures 1*(a)*–*(c)*. These instabilities result in ascending and descending fingers across the initial contact line in a symmetric way [3, 4, 5].

NUMERICAL SIMULATIONS

To gain further insight into the nonlinear dynamics of the various differential diffusive instabilities and to make comparisons with our experimental observations, we perform nonlinear simulations. This would indeed permit us to explore a wider parameter space, due to the time constraints inherent in laboratory-scale experiments. A finite-volume method has been adopted to simulate the system of equations [6]. Extensive validations tests have been performed using comparisons



(a) *(b)* *(c)* *(d)*

Figure 2. Buoyancy-driven instabilities at the miscible interface between two fluids, visualized using the concentration map of A: *(a)* Rayleigh–Taylor $(R, \delta) = (0.5, 1)$, *(b)* Double-diffusion $(R, \delta) = (1.16, 3.67)$, *(c)* mixed-mode instability (DLC + RT) $(R, \delta) = (0.8, 0.3)$. *(d)* Mixing length as a function of time for various parameter configurations corresponding to DD instability. The red line corresponds to a slope of 0.5 as the mixing length grows as $t^{0.5}$ in a diffusive regime.

with theoretical linear stability analyses, and our results for our growth rate and wavelength are in excellent agreement with them.

The different buoyancy-driven convective instabilities are shown in figures 2*(a)*–*(c)*. As expected, the convective patterns evolve the same way on both sides of the initial interface. Figure 2*(d)* shows the evolution of mixing length as a function of time for various parameter configurations corresponding to DD instability. These observations are in congruence with those presented in [4]. For the case of $\delta = 3$ and $R = 2$, they had observed that the mixing length indeed grows as $t^{0.5}$. Decreasing the value of R has a destabilizing effect as the stabilizing density ratio is then decreasing, resulting in larger values of the mixing length.

Currently comparisons on the wavelength and the growth rate of the mixing length under various parametric regimes are being done to establish a connection between our experimental and numerical results. These would be presented in detail during the conference.

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